EXISTING PROBLEM AND FUTURE SCOPE OF DYNAMIC RESPONSEANALYSIS OF GROUND

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1 INTRODUCTION

Geotechnical engineer's circumstances around dynamic response analyses have changed rapidly in recent years in three meanings.

Firstly, geotechnical engineers got powerful tools. Because of the development of a computer technology, he can make a huge calculation, which was a dream 10 years ago, on a desk by means of a personal computer. At the same time, many constitutive models have been proposed and improved especially related to soil liquefaction of sand, which is one of the main interests for the engineer, and considerable number of computer codes have been developed.

Secondly, requirement on the dynamic response analysis has changed. In Japan, design specifications are going to change to consider, so called, level 2-ground shaking especially after the 1995 Hyogoken-nambu (Kobe) earthquake. The level 2-ground shaking is the largest ground shaking that will be expected at the site in future[1]. Therefore, it is very large compared with the ground motion defined in conventional design specifications. Under this strong ground shaking, structures are sometimes difficult to be kept undamaged. Then the concept of performance-based design is going to be employed in the design specification. Under the performance-based design, partial damage to structures is allowed if required performance of the structure is maintained. In this situation, structural engineers sometimes request geotechnical engineers to output the behavior of ground up to failure or even more.

Finally, since dynamic response analysis becomes easy to handle, many requirements appears; result of the dynamic response analysis are going to be used in various way. In the design of a pile, for example, a structural engineer needs displacement distribution of the ground. If he is going to design a machine plant, response in high frequency component is required as output.

In the old days, peak ground acceleration and acceleration time history was sufficient as an output of the dynamic response analysis, which belongs to a category that is the easiest to predict in the dynamic response analysis. Evaluation of displacement is much more difficult, especially when soil is close to failure or after failure.

In order to respond these variety requirements, dynamic response analysis should be accurate more than previous.

There are many steps procedure to obtain final results of the dynamic response analysis as shown in Figure 1. More or less error will be produced at each step. Since final error of the analysis is expressed by a kind of sum of the error at each step, it cannot become smaller than the maximum error at each procedure. Therefore, the most effective method to increase the accuracy of the dynamic response analysis is to reduce the largest error. At the same time, it is also important to reduce the error at each step although reduction of error in only one step may not bring significant improvement of the result.

In this paper, the author intends to show existing problems of the dynamic response analysis of ground at each procedure to obtain final result for a future improvement of the research briefly, especially in focusing on accuracy.

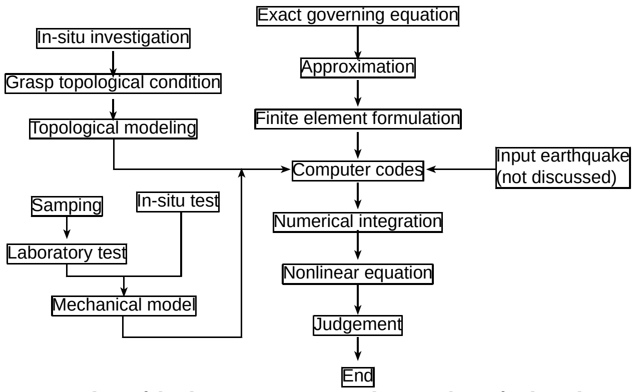


Figure 1 Flow of the dynamic response analysis to obtain final result

2 GOVERNING EQUATION

Soil is a mixture of soil particles that forms skeleton structure, water and gas. The governing equation, therefore, should be described by these three constituents. This kind of multi-phase equation exists (e. g. ref 2), and was used in the simple case study, but it has not been used in practical problems. A two-phase treatment is commonly used in practical use at present. Biot's equation, which is identical to the multi-phase governing equation under the saturated condition, is the most frequently used governing equation. In order to use Biot's equation, the ground is modeled to be either dry or saturated state; partially saturated state cannot be considered. This indicates that change of water table due to excess porewater pressure dissipation, for example, cannot be considered. There is a method to treat partially saturated state within the category of Biot's equation, and is used in the seepage analysis[3], but not in the dynamic problem.

The governing equation for the dynamic behavior of ground is described by displacement of the soil skeleton u, displacement of water U and porewater pressure p based on the Biot's equation in the most accurate form, which is called u-U-p formulation. Apparent displacement of water relative to soil skeleton $w=(u-U)/w$ may be used instead of U, in which case the governing equation is called u-w-p formulation. There are several approximated formulations to reduce number of independent variables. Flow to derive these formulations is summarized in Figure 2, including static and quasi-static formulations. Error caused by the approximation is estimated to be less than several percents in the ordinary situation of the dynamic response analysis of sand including liquefaction analysis, except an undrained assumption ($u=U$ or $w=0$).

The undrained assumption has been frequently used in the liquefaction analysis under the consideration that duration of an

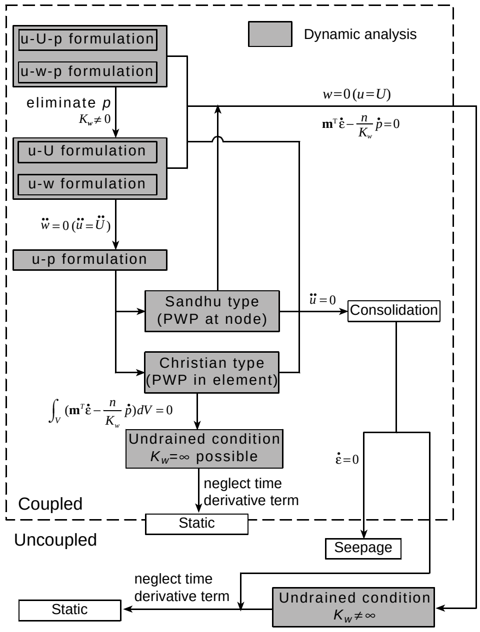


Figure 2 Biot's equation: accurate and approximate formulation including consolidation and static formulations.

earthquake is too short for the excess porewater pressure to drain. This assumption makes computer coding much easier because governing equation yields one phase equation. Therefore, it has been used in many liquefaction analyses. Change of the excess porewater pressure, however, is not always negligibly small because product of the bulk modulus of water and volume change of the porewater can be finite order as the bulk modulus of water is very large even if the total amount of flow of porewater is very small. It sometimes causes error with 10% or more in displacement; an example is shown in Figure 3[4)]. In particular, there are cases in which consideration of drainage are important. An example is an analysis of gravity caisson quay wall. It is usual to place gravel at the back of the caisson quay wall. Since dynamic pressure acting on the back of the caisson is one of the important factors in discussing the stability of the quay wall and permeability of the gravel is very large, consideration of porewater pressure redistribution becomes necessary.

There is another approximation, which is called a total stress analysis. Definition of the term "total stress analysis" is used when excess porewater pressure generation is neglected. Since there becomes no water flow, the ground is modeled into one-phase media. Change of elastic and nonlinear properties caused by the change of total stress is sometimes considered and sometimes not. Equivalent linear analysis, which will be discussed later, uses the latter assumption, and truly nonlinear analysis (nonlinear analysis, hereafter) usually uses the former assumption. Error induced in the total stress analysis has been reported in literatures. It is larger in the liquefaction problem (e.g., ref. 5), whereas not so predominant when excess porewater pressure generation is not important. Error of the analysis is the largest when material property is kept unchanged which is assumed in the equivalent linear analysis. In addition, equivalent linear analysis has another error, which will also be discussed later.

In addition to the error derived from the governing equation, application to finite element also creates error. General discussion on the finite element method will not be made because since there are so many textbooks. Instead of it, only one example is introduced which will be occur commonly in the analysis of ground under earthquakes.

As one can easily understood from the fact that undrained assumption is frequently used in the analysis of ground, deformation of the ground during earthquake is nearly no

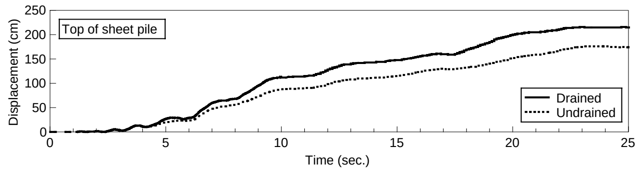


Figure 3 Horizontal displacement at the backfill of the ground under drained and undrained conditions. (modified from ref. 6)

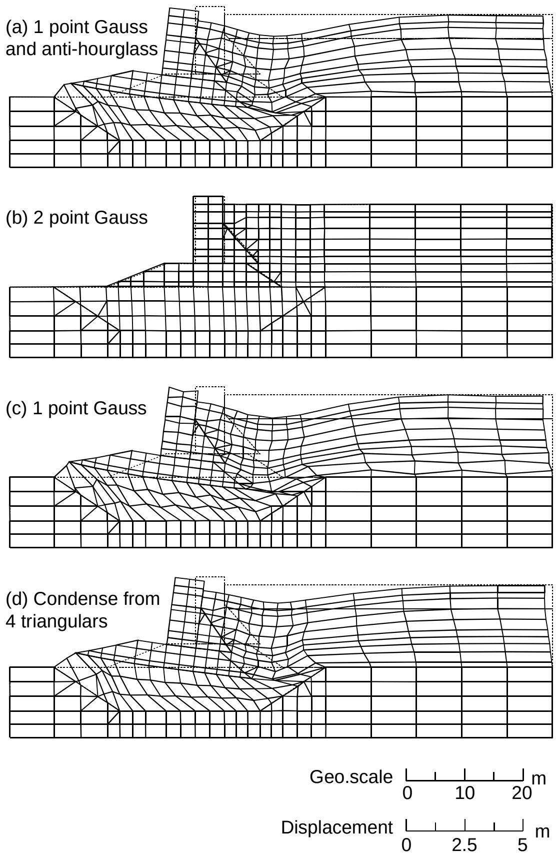


Figure 4 Shear locking, hourglass instability and its solution

volume change behavior, i.e., apparent Poisson's ratio is close to 0.5. In this situation, ordinary method to build an element stiffness matrix sometimes locks deformation of the element; deformation is significantly underestimated, which is called shear locking. Reduced integration is known to be effective to avoid shear locking. However, it may cause another problem on stability, which is called hourglass instability. It is also known that, so-called, anti-hourglass stiffness can avoid hourglass instability. As seen a fairly complicated procedure is necessary in order to avoid shear locking in the analysis of ground compared with the ordinary FE analysis.

An example is shown in Figure 4[7)] in which four deformed figures are drawn in the same scale. Figure 4(a) is the most accurate result in this problem as explained later. Figure 4(b) is a result by the use of 2x2 points Gauss-Legendre integral for quadrilateral element, which is commonly used method to build element stiffness matrix in the ordinary finite element analysis. Displacement is much smaller than that in Figure 4(a), which is a typical appearance of the shear locking.

Displacement in Figure 4(c) is obtained by the reduced integration (one-point Gauss-Legendre integration). Displacement becomes much larger than previous. It is the same order with Figure 4(a), but displacement is not smooth in the surface layer and in the replaced sand under the caisson, which is a typical appearance of hourglass instability. The name "hourglass" comes from the deformed shape; deformed shape of two adjacent elements seems like an hourglass. Displacement in Figure 4(a) is obtained by reduced integral and anti-hourglass stiffness[8], which is supposed to be the most accurate result in this problem. Compared with Figure 4 (a), displacement in Figure 4 (b), obtained by ordinary FE technique, is too small that the engineer cannot accept this result. On the other hand, displacement in Figure (c) is nearly similar with the one in Figure 4 (a), except that there appear hourglass instability deformations.

Finally, Figure 4 (d) is obtained by another method to avoid shear locking and hourglass instability. Each quadrilateral element is divided into four triangular elements and computed four element stiffness matrices are condensed into one element stiffness matrix. Since shear locking does not occur in triangular element, problem discussed above does not occur. If, triangular element is free from shear locking, it seems better to use triangular element instead of quadrilateral element. However, this may not be good because, in the problem close to failure, error by triangular element is known to be larger than that by quadrilateral element. One can recognize the difference between quadrilateral element and triangular element from Figure 4 (a) and (d); deformations in Figure 4 (a) and (d) are similar but not identical.

Since the procedure shown above is not a common situation in the ordinary structural analysis, computer codes do not always install these functions; error can be seen from the difference between Figure 4 (c) and (d) may not be avoidable.

There are several other approaches. Boundary element method may be good in treating an infinite region, but consideration of nonlinear behavior, which is essential in the dynamic response analysis of ground, cannot be considered. Finite difference method has difficulty in treating different materials, therefore hardly used in the dynamic response analysis in practice. Discrete element method (DEM) has been shown a powerful tool in recognizing the behavior of soil because one can see the behavior of soil particle[9]. It helps to recognize the behavior qualitatively, but quantitative evaluation seems difficult.

3 TREATMENT OF TIME

The governing equation is a simultaneous partial differential equation with respect to time and space, among which time is treated in this section, and space will be discussed in the next chapter. The governing equation is solved either in the time domain or in the frequency domain.

3.1 *Time domain analysis*

Step-by-step time integration scheme is used in the time domain analysis. There are many schemes, among which Newmark's β method, Wilson' θ method and central difference method are the most frequently used methods in the dynamic response analysis. There is no absolutely accurate solution in the conventional dynamic response analysis because input acceleration is specified only at scattered times (in other words, the problem is not defined completely). Each time integration scheme uses different interpolation between times where value is specified. Newmark's β method (β=1/4), for example, assumed that acceleration response is constant during the time increment Δt, whereas it is assumed to change linear within $\theta\Delta t$ in the Wilson's θ method, and displacement is assumed to change by the second order equation in the central difference method. We cannot call the difference as "error" because of the above-mentioned reason, but the result is different depending on the integration scheme.

Stability is one of the important issues to choose the time integration scheme. There are two meaning of the term "stability" in the practical use.

The one is a stability of the numerical integration scheme, which is not discussed in detail here because there are many textbooks that treat it. For example, absolutely stable condition (stable regardless time increment) can be obtained if $\beta\geq0.25$ for

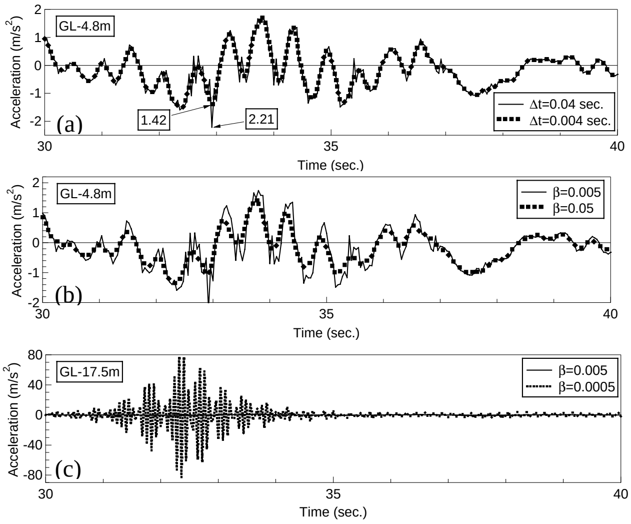


Figure 5 Example of pulse response. (a) Pulse wave at about 33 seconds (b) Elimination of pulse by larger damping (c) Many pulse waves by smaller damping

Newmark's β method and $\theta\geq1.37$ for Wilson's θ method. The engineer should consider, however, that too large β and θ decreases accuracy very much. It is also noted that stability may not be guaranteed in the nonlinear analysis even if above-mentioned conditions hold; examples that numerical integration diverged are shown in several technical papers and even in textbooks (e.g. ref. 10).

Another is a stability that occurs in the process to solve nonlinear simultaneous equation, by which reasonable solution cannot be obtained. If a perfect technique is used, probably this kind of problem may not occur, but, in the actual situation, it sometimes occurs especially in the liquefaction analysis. One of the methods to avoid it is to use large damping; an example is shown in Figure 5 later. For example, Wilson's θ method causes much damping than Newmark's β method; therefore, more stable in the latter sense although accuracy of the Wilson's method is less than that of the Newmark's method. Another damping (velocity proportional damping) is also employed to reduce the risk of stability. Sometimes, several percents of damping ratio is used in the predominant period as stiffness proportional damping, which seems unrealistically large damping. There may be many reasons why this kind of instability occurs, but they were hardly or never reported in the technical papers; only succeeded results have been reported. Only one possibility is shown in the following.

In the conventional method, input acceleration increments are applied as impulse forces in each time increment. If there is no damping and no stiffness, response acceleration will show pulse shape response due to the impulse input. In the ordinary situation, since impulse force is absorbed by the damping and stiffness term, we hardly see pulse shape wave. However, if stiffness becomes too small, which frequently occurs in the liquefaction analysis and nonlinear analysis close to failure, or input is too large, pulse wave appears. An example of pulse response is shown in Figure 5(a). Pulse appears at about 33 seconds in the calculation with Δt=0.04 sec., which result in 2.21 m/s^2 peak acceleration. It disappears when time increment becomes Δt=0.004 sec., resulting in 1.42 m/s^2 peak acceleration. It is also recognized that this pulse response hardly affects the overall behavior and in many other cases as far as the author knows; it disappear gradually like a damped free vibration. However, if pulse is too large, it begins to affect the response of nearby element, and sometimes causes instability of the problem. It is also noted that peak acceleration is not reliable in this case.

Instead of using a smaller time increment, pulse response can be eliminated by employing large damping as shown in Figure 5(b). Two waveforms are similar but there are some differences.

In other words, it can be said that calculation using large damping succeeded to eliminate pulse wave, but accuracy is lost. It is also noted that, the engineer will not feel that there are error in his calculation when larger damping is used if he see only this result without comparison with more accurate result.

On the contrary, if damping decreases many pulse waves appear as can be seen in Figure 5(c). Note the difference of ordinate; pulse under smaller damping is too large that accurate result seems like a horizontal line. Through the case study from Figure 5(a) to Figure 5(c), one can recognize that there are relevant ranges for damping from the point of view of the stability of analysis; larger damping looses accuracy whereas smaller damping also loose accuracy because of pulse wave that may become instability problem.

3.2 *Frequency domain analysis*

Differentiation with respect to time and space are separated by means of Fourier transfer in time. Since material property should be kept constant in whole duration of an earthquake, so called equivalent linear method is the only method to consider nonlinear behavior of soil. It is noted, however, so-called equivalent method is not an equivalent method in mathematical meaning, but just an approximated method. Accuracy, therefore, is less than nonlinear method. There are, however, several advantages in the frequency domain analysis even if accuracy is less.

Firstly, mathematical treatment becomes easier; exact solution of the differential equation may be obtained or governing equation becomes simpler even if exact solution cannot be obtained. Secondly, frequency proportional property can be easily considered. A typical example is surface wave propagation and damping due to scattering. Ground compliance is also frequency dependent. It is difficult to consider them in the time marching analysis.

Therefore, both time domain analysis and frequency domain analysis should be used properly considering their advantage and disadvantage. Accuracy of equivalent linear analysis will be discussed later.

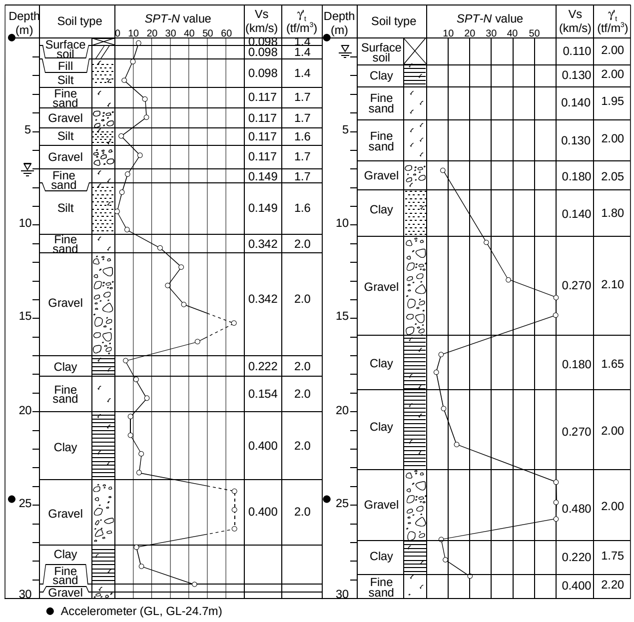
4 TREATMENT OF SPACE

A two-steps procedure is necessary related to the treatment of space in order to use the dynamic response analysis. The first one is to grasp the topographical configuration and the second to model it to match computer codes. It is not be discussed here because there may be a volume of one book to discuss it. Two examples are shown related to the latter topic[11].

Elastic modulus should be measured at the field because there is huge error in the elastic modulus measured in the laboratory[12]. They are measured by P-S logging. Both downhole method and suspension method are used frequently.

A typical difficulty in evaluating them is shown in Figure 6. This figure is a soil profile at Technical Research Center, Kansai Electric Power Co.[13], where vertical array strong seismic records were obtained during the 1995 Hyogoken-nambu earthquake. Borehole investigations and PS loggings were conducted two times at this site. The first one was conducted before the earthquake by the downhole method and the other after the earthquake by the suspension method.

Yoshida et al.[14] pointed out 2 questions on the first data when analyzing this site just after the earthquake, among which the second question is important in the discussion here. S-wave velocities are the same to be 117 m/s from GL-3.6 to GL-7 m although the subsoil consists of different materials such as fine sand, gravel, and silt layers. Their point is that S-wave velocities in the gravel layers should be larger, and those in the silt layers should be smaller than reported when looking at the SPT data and based on the general information nearby this site. They, however, did not modify shear wave velocities when conducting



(a) Before earthquake (downhole) (b) After earthquake (suspension)
Figure 6. Soil profiles based on borehole investigations before and after the earthquake (data below GL-30m is not shown here)

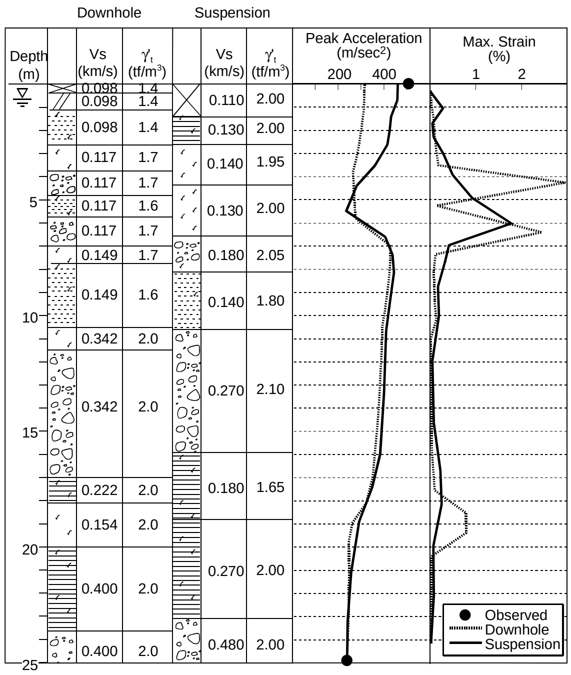
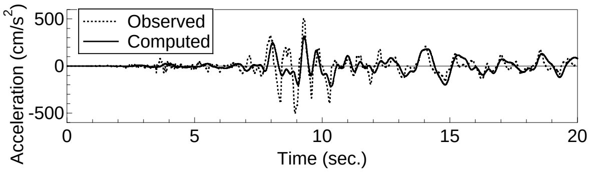
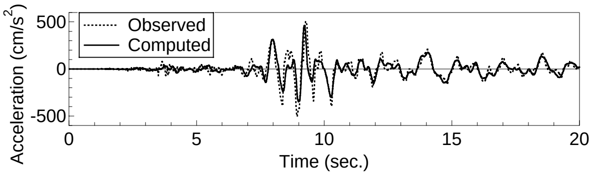


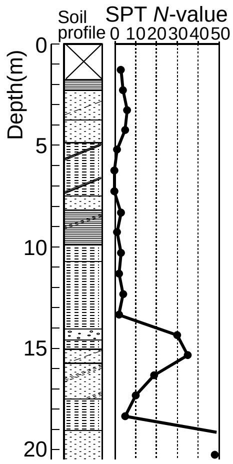
Figure 7. Peak acceleration and maximum strain distribution



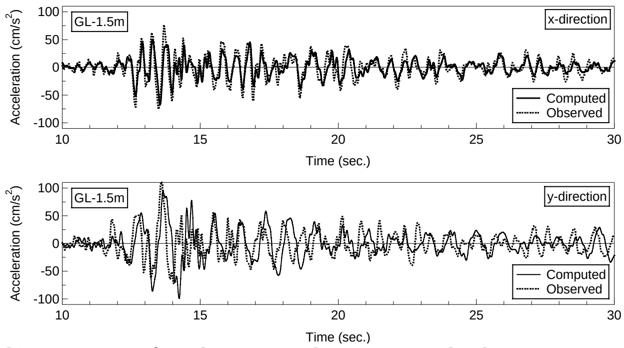
(a) Before the earthquake



(b) After the earthquake
Figure 8. Comparison of time histories at the ground surface



(a) Soil profiles



(b) Comparison of acceleration time history in x- and y-direction

Figure 9 Soil profiles and acceleration at Tokyo Bay area

the dynamic response analysis because they are in-situ measured data. Shear wave velocities by a downhole method are frequently set constant within several layers as shown above and the engineer usually does not expect to change measured data; therefore, their analytical procedure is common in the engineering practice. Peak responses by the equivalent linear analysis that uses this value are shown in Figure 7 by dotted line as designated by Downhole, and acceleration time histories are compared in Figure 8. Peak acceleration is significantly underestimated at the ground surface as seen in a dashed line in Figure 7 and solid line in Figure 8(a). The reason of this underestimation is clear; severe nonlinear behavior occurred at the gravel layer. Gravel shows nonlinear behavior at smaller strains than sand and silt, therefore, constant shear wave assumption shown in Figure 6 results in the early nonlinear behavior in the gravel layer. This underestimation of the peak acceleration proves that the shear wave velocity in Figure 6 (a) is not relevant although they are in-situ measured value.

Result of the in-situ test after the earthquake is shown in Figure 6(b), in which PS logging was conducted by the suspension method. Ability of a suspension method to catch local changes in wave velocities is reported in several literatures. Soil profiles and SPT *N*-value are similar to the result before the earthquake but significant differences appear in V_s and unit weight γ_t. The shear wave velocity of the gravel is evaluated higher than that of the sand and clay as pointed out before. The result of the equivalent linear analysis is shown in Figure 7 and Figure 8(b). It is very much improved from the previous analysis. The peak acceleration is, however, still smaller than the observed record, which is against the knowledge that an equivalent linear analysis overestimates peak acceleration under a strong ground shaking, which will be discussed later in chapter 6. The reason of this underestimation is again clear when looking at the peak response distribution in Figure 7. Maximum strain of 1.8 % is observed at GL-6 m. Severe nonlinear behavior occurred in this layer, resulting in sudden decrease of peak acceleration at this layer.

Shear wave velocity in this layer is 130 m/s, which is smaller than those above and below this layer; a weak layer is sandwiched. In this situation, multiple reflections occur at the top and bottom boundary of this layer, which results in the concentration of the kinematic energy in this layer, hence significant nonlinear behavior. This energy concentration will actually occur if this site is a horizontally layered deposit, but, as can be seen in Figure 6, this site is not a horizontally layered deposit. Therefore, energy concentration in this particular layer is difficult to occur as can be seen in the one-dimensional analysis. For example, increase of V_s in this layer from local wave velocity to the average value will improve the accuracy of the analysis.

This example indicates that a suspension method gives more accurate wave velocities than a downhole method. It gives, however, the data along the particular hole, therefore may not be a relevant or representative value to be used for the dynamic response analysis. This also shows difficulty in determining a soil profiles and elastic moduli for the dynamic response analysis.

Another problem is also reported by the author[11]. Figure 9(a) shows soil profile at a site in Tokyo Bay area where vertical array observation was conducted. Analysis is conducted by the equivalent linear method under the 1987 Chibaken-toho-oki earthquake. Time histories at GL-1.5 m are compared with observed record in Figure 9(b). The agreement in x-direction is fairly good. On the other hand, the simulation in y-direction is less accurate than that in x-direction. If the ground is level, agreement in both directions should be the same order. Therefore, we may be able to say that it was lucky to obtain a good agreement in x-direction. If one wants to obtain good results for both x- and y-directions, one may need to conduct one-dimensional analyses in x- and y-directions under different soil profiles or multi-dimensional analysis.

These examples show importance to ensure the accuracy or reasonability of the model. The author does not know a good method, but back analysis of the vertical array record under small earthquake or evaluation by means of microtremour measurement seems attractive.

Discussion in this paper is limited for one-dimensional analysis. Two- and three-dimensional analyses must have another problems in addition to the problems here.

5 MATERIAL PROPERTY AND MODELING

Same as modeling of space, a two-steps procedure is necessary in modeling the material property. The first one is to recognize and to describe the nonlinear property of soil and the second to model it for constitutive models.

Existing problems for the first procedure is also discussed by the author in detail in ref. 11 and. 15. The key issue was that the conventional laboratory test is not a relevant element test for the dynamic response analysis of ground at large strains, which will be introduced briefly.

If the material is steel, for example, several indices such as elastic moduli, yield stress, and strain hardening strain, etc. are sufficient to reproduce the behavior of the steel element with sufficient accuracy. In the case of soil, however, the behavior is so complicated that it is impossible or very difficult to find indices that can reproduce the whole behavior by single test. Therefore, laboratory test has been conducted to retrieve the behavior necessary for the analysis.

In our field, dynamic deformation characteristics test and liquefaction test are the most commonly used test. Back to the past, the former was sufficient for the equivalent linear analysis, and the latter was sufficient for identifying the onset of soil liquefaction. However, these tests are now insufficient to respond grown various requirements on the dynamic response analysis discussed in the introduction of this paper.

In order to obtain the dynamic deformation characteristics, 11 cycles of loading are applied at each stage, and stabilized

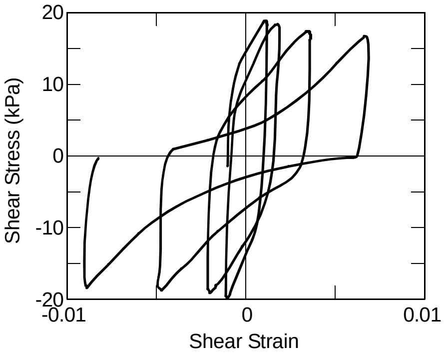


Figure 10 Stress-strain curves at large strains. Hysteresis curves does not become stable at large strains.

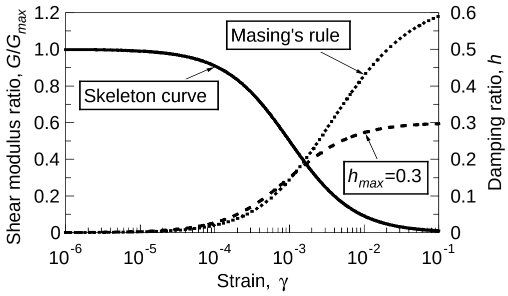


Figure 11 Error of damping ratio by Masing's rule

hysteresis loop is used to compute shear modulus G and damping ratio h as a function with respect to shear strain amplitude γ according to JGS standard[16]. This procedure works up to strains of 0.1 % or a little more. However, if shear strain increases more, the hysteresis loop no more becomes stable, but shear strain begins to increases as loading cycles as shown in Figure 10. Therefore, a new test method should be developed to recognize the ground in this strain range.

The same discussion can be made for the liquefaction of sand. The liquefaction strength curve, i. e., the relationships between shear stress amplitude (frequently normalized by the initial effective confining stress) and number of cycles causing liquefaction, was sufficient if onset of soil liquefaction is of primary interest. However, if the engineer wants to analyze the behavior of ground after soil liquefaction, no data is given by the conventional test.

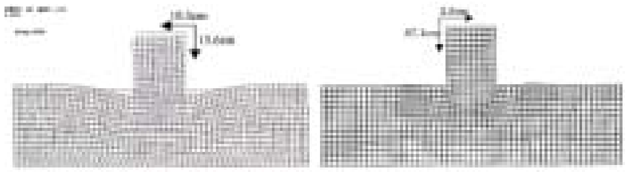
It is also noted that dynamic deformation characteristics test covers strains up to 0.1 % or a little more, but not more than 1%. On the other hand, liquefaction strength test deals with several percents strain. Therefore, data on the behavior of sand is missing even before liquefaction.

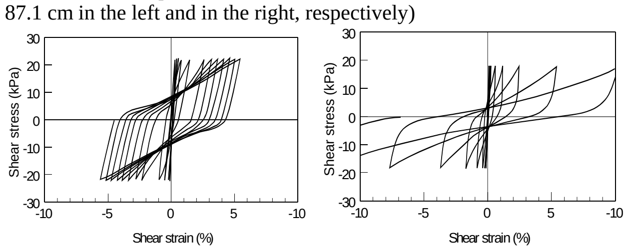
If the engineer does not have sufficient data on soil behavior, he cannot expect good evaluation of the behavior of ground by the dynamic response analysis.

Next, let assume that material property is grasped completely, then the engineer should move the next stage. i.e., to model it to fit the constitutive model that the computer code prepares. There are many issues to be discussed on this process, among which two topics will be discussed in this paper.

(1) Masing's rule

In many constitutive models, the behavior of soil is described by two situations. A backbone curve or a skeleton curve expresses the behavior under initial or virgin loading, whereas hysteresis curve expresses the behavior after unloading takes place. The Masing's rule is a rule to make a hysteresis curve from a skeleton curve. If the skeleton curve is expressed by $\tau = f(\gamma)$, the then hysteresis curve is obtained by $(\tau - \tau_R)/2 = f(|\gamma - \gamma_R|/2)$, where τ_R and γ_R are stress and strain at the recent unloading point. They will be replaced by hardening parameter and plastic strain for multi-dimensional analysis.


(a) Deformed shape (Note that scale is different; settlement is 15.6 and 87.1 cm in the left and in the right, respectively)



(b) Example of stress-strain curve

Figure 12 Comparison of settlement of tank and stress-strain curve.

The test result shows that this rule does not hold, but, probably because of the simple mathematical treatment, it has been used in many constitutive models.

For example, Hardin and Drnevich[17] proposed design equations for G-γ and h-γ curve as

$$G = \frac{G_{max}}{1+\gamma/\gamma_r} \qquad \text{or} \qquad \tau = \frac{G_{max}\gamma}{1+\gamma/\gamma_r} \tag{1a}$$

$$h = h_{max}(1 - G/G_{max}) = \frac{h_{max}}{1+\gamma/\gamma_r} \tag{1b}$$

where γ_r is a reference strain.

On the other hand, application of Masing's rule in Eq. (1a) yields

$$h = \frac{4}{\pi}\left(1+\frac{\gamma_r}{\gamma}\right)\left[1-\frac{\gamma_r}{\gamma}\ln\left(1+\frac{\gamma}{\gamma_r}\right)\right]-\frac{2}{\pi} \tag{2}$$

which is quite different form from Eq. (1b). However, as shown in Figure 11, the difference is small when strain is less than or equals to medium strain, which is probably agrees with the strain range where conventional analysis has been conducted. This may be another reason why Masing's rule has been used. As clearly seen in Figure 11, however, error in damping ratio is very large at large strains.

(2) Accuracy of constitutive model for liquefaction analysis

A committee of Japan Society of Civil Engineering is now undergoing simultaneous analyses on settlement of a storage tank. Figure 12 shows two case studies among the undergoing work.

Stress-strain curves obtained in the simulation of the liquefaction strength test are also shown in the figure. It is noted that both models simulates liquefaction strength curve almost perfectly. Therefore, in the conventional sense, these two models show the same behavior because only the liquefaction strength curve is given as a target to ensure the accuracy of the constitutive model. As can be seen in the figure, however, calculated settlement differs significantly.

At present, we believe that the difference comes from the difference of the slope of the stress-strain curve. Since settlement is an irreversible phenomenon, it accumulates in each cycles of loading. Therefore, stress-strain curve that has plateau or small stiffness portion produces more settlement than the one with larger stiffness although average stiffness is the same. In other word, expression slope in the middle of the hysteresis loop brought the difference of the settlement.

This example shows that constitutive models are required to reproduce not only liquefaction strength but also slopes of the stress-strain curve; expression of the hysteretic behaviors by damping ratio as dynamic deformation characteristics is far from sufficient if the engineer is going to predict settlement of structures.

6 EQUIVALENT LINEAR METHOD

Unlike the name "equivalent", equivalent linear method represented by SHAKE is just an approximate method. Deficiencies of this method are discussed in Ref. 18, and will be introduced later.

Theoretically speaking, there is no doubt that nonlinear analysis is more accurate than equivalent linear method. However, equivalent linear method has several advantages if it is applied in the frequency domain.

A very important advantage of an equivalent linear method is a deconvolution analysis by which the incident wave to the engineering seismic base layer or any other layers can be computed from the earthquake record at the ground surface or any other point when multiple reflection theory such as SHAKE is used. On the other hand, nonlinear analysis allows only convolution analysis. Another advantage is an ability to consider frequency dependent characteristics such as damping due to the scattering of waves and surface wave propagation. These can be easily considered in the frequency domain analysis, but not in the time domain analysis used in nonlinear methods. In addition, nonlinear method has problems in reproducing high frequency components partly because of the numerical damping induced in the numerical integration scheme and partly because artificial damping such as stiffness proportional damping that suppresses high frequency behavior as discussed in the preceding.

Therefore, both nonlinear analysis and equivalent linear analysis should be used to compensate their deficiencies. If so, equivalent linear analysis is better to produce smaller error.

There are two deficiencies in the equivalent linear analysis beside the well-known deficiency that is cannot consider change of material property during earthquake.

The first one is overestimation of shear stress under large earthquake. Figure 13 shows the mechanism of the overestimation schematically. Let solid line is a specified stress-strain curve. Then equivalent linear method uses a linear stress-strain line OAC, in which A is located on the skeleton curve corresponding to the effective strain γ_{eff} which is smaller than maximum strain γ_{max}. Therefore, maximum stress that corresponds to the point C is always overestimated. This overestimation of shear stress may not produce significant error at small to medium ground motion, but becomes significant effect at large strains, because material can carry shear stress larger than actual shear strength. If shear stress is overestimated, peak acceleration is also overestimated.

Another deficiency of the equivalent linear method can be easily seen by comparing the amplification factor, i.e., ratio of Fourier amplitudes of the accelerations. An example is shown in Figure 14[19], in which amplification computed from the vertical array records at Tokyo Bay area is compared with SHAKE. One can recognize that amplification by SHAKE is much less than the observed amplification in the frequency larger than several Hz. The mechanism of this underestimation seems clear. Since SHAKE evaluates the stiffness and damping based on the effective strain, larger damping and smaller modulus are used even at the high frequency behavior in which the amplitude is much smaller than the effective strain. The phenomena shown in Figure 14 are commonly observed in the analysis by the conventional equivalent linear analysis.

It seems that this underestimation may not be significant when lower frequency behavior or peak acceleration is of interest. This is, however, not always true.

The problem occurs in the deconvolution analysis under a strong ground shaking. Amplification smaller than unity indicates that shaking at the base layer is larger than the shaking at the ground surface. In other words, acceleration becomes larger in the vertical downward direction. This inverse amplification may result in unrealistically strong incident wave, and sometimes, analysis does not converge but diverge. This tendency becomes predominant at strong ground shaking. As discussed in the preceding, the deconvolution analysis is one of the important features of SHAKE; therefore, this is a critical defect. In order to avoid this problem, one need to use a small α value or need to cut the high frequency component, both of which obviously accelerate error of the analysis.

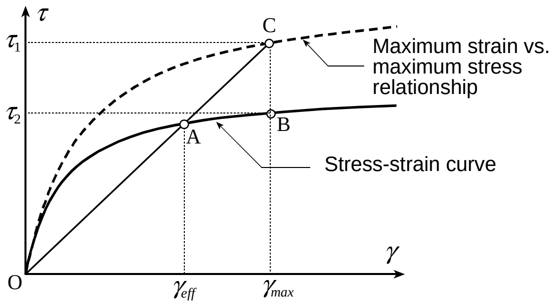


Figure 13 Schematic figure showing the mechanism of the overestimation of the shear stress by the equivalent linear method.

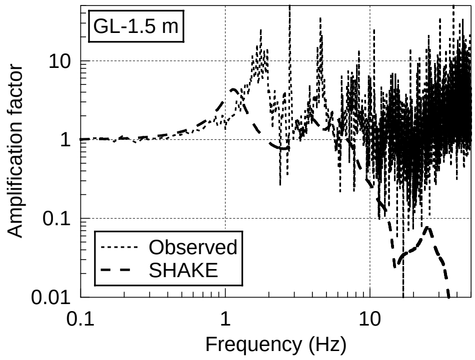


Figure 14 Underestimation of amplification factor by SHAKE

Two deficiencies described above have opposite effects on the peak acceleration, but come from the same cause as easily recognized from the previous discussion, i.e., the method of determining the shear modulus and damping ratio from the effective strains by the equation

$$\gamma_{eff} = \alpha\gamma_{max} \tag{3}$$

In order to improve the first deficiency, i.e., overestimation of the peak acceleration, the value of α should become larger. On the other hand, in order to improve the second deficiency, i.e., underestimation of the amplification in high frequency region, it should become smaller. Therefore, one cannot solve these problems at one time. This is a dilemma to determine the value of α in the conventional equivalent linear method.

The idea to overcome these two deficiencies is proposed by Yoshida et al.[18] by defining the effective strain as a function with respect to circular frequency. The result is not introduced here; see ref. 18 if necessary.

7 DAMPING

Damping term, i.e., the term proportional to velocity is included in the equation of motion without doubt. However, mechanism of the damping term is not clear in many cases although it is, of course, sure that vibration terminates some time indicating the existence of damping. Besides true damping, damping term has been used in variety way, for example, to compensate things that are not considered in the analysis or to adjust the result; uncertainty of the mechanism of damping makes easy to use it in various ways. For example, several percents of material damping is sometimes used for the structural element in order to represent radiation damping in rigid base analysis.

Two topics will be introduced here.

(1) Damping caused by wave scattering

The ground is modeled into homogeneous material. However, actual ground is not homogeneous even in a same layer. This indicates that, unlike the assumption that waves propagate straight in the surface layer, waves do not propagate straight but scatters. From the point of vies of the homogeneous ground, this behavior looks as if the ground has damping, which is called damping caused by wave scattering, and will be called scattering damping hereafter for simplicity in this paper. Order of scattering damping is several percents at the predominant period; therefore, it cannot be neglected.

Based on the back analysis of the observations, many regression equations have been proposed. A typical equation as a function with respect to frequency *f* is expressed in a form

$$h = af^{-b} \tag{3}$$

where *a* and *b* are positive numbers. Looking at the proposed equations in the past researches, observed damping value scatters very much. This is natural if scattering damping is caused by the reasons above because degrees of inhomogeneousity depend on site. This means that, unless scattering damping is measured in-situ, one cannot expect error smaller than the error that scattering damping has.

The effect of nonlinearity to the scattering damping is also not clear. If inhomogeneous material is deformed, the weaker portion has a tendency to deform more than the stronger portion. Therefore, scattering damping is supposed to increase more when nonlinear behavior occurs. However, there is little research dealing with it.

(2) Rayleigh damping

Rayleigh damping is used in all computer codes in the nonlinear dynamic response analysis of ground as far as the author knows. A damping matrix [*C*] is expressed as a linear combination of mass matrix [*M*] and stiffness matrix [*K*] as

$$[C] = \alpha[M] + \beta[K] \tag{5}$$

where α and β are parameters. The modal damping h_i is expressed as

$$h_i = \frac{\alpha}{2\omega_i} + \frac{\beta\omega_i}{2} \tag{6}$$

where ω_i is circular frequency at i-th mode. The stiffness proportional term is important for the stability of the numerical analysis as described in the preceding; therefore, only this term is discussed here.

Error occurs by large damping is already discussed in the preceding. It is easily recognized that error is larger at high frequency component because response is over-damped. This can be understood qualitatively from the shape of the equation, but there is no tool to evaluate the error quantitatively.

The author improved conventional analyses by installing additional function to visualize the effect. It is made in both time domain analysis as well as frequency domain analysis.

In the time domain analysis, modal damping can be specified in arbitrary way. Derivation is not shown in this paper, but final expression for the lumped mass system yields as

$$[C] = [M][\xi]\left[2h_j\omega_j\right]\left[1/m_j\right][\xi]^{\mathrm{T}}[M] \tag{7}$$

where [ζ] is a matrix composed of eigen vectors, [$2h_i\omega_i$] is a diagonal matrix whose component is $2h_i\omega_i$, and [$1/m_j$] is a diagonal matrix whose component is inverse of mass matrix. The frequency independent damping can be obtained by setting all h_i's constant.

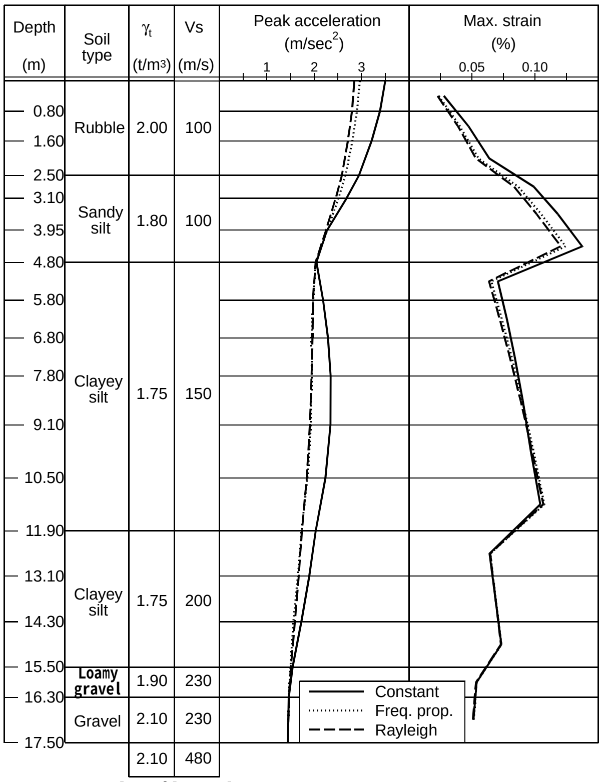


Figure 15 Soil profiles and maximum response

On the other hand, frequency dependent damping

$$h = \frac{\alpha}{2\omega} + \frac{\beta\omega}{2} \tag{8}$$

is incorporated in the frequency domain analysis. It is noted that this equation is similar to Eq. 6, but one should be careful that Eq. 6 indicates damping constant depends on vibration mode (therefore only discrete values) whereas Eq. 7 is a continuous function with respect to circular frequency. Therefore, both equations are not identical. Considering that amplification is the largest at each mode, we can expect that both results give almost identical result.

A case study is conducted to see how Rayleigh damping affect the result of the dynamic analysis. A site at the downtown Tokyo is chosen and level 2 artificial earthquake motion is applied. In order to retrieve the effect of damping, elastic behavior is assumed for both time domain and frequency domain analyses. Two frequency domain analyses and two time domain analyses are conducted. In each domain analysis, one analysis assumed constant damping of 2%, and frequency dependent damping as explained above is used in another analyses. In the case of frequency proportional damping, 2% damping is used at the predominant period of the ground (0.418 sec.).

Figure 15 shows soil profiles and maximum response. Two constant damping cases show almost the same response; therefore, only one line is drawn as designated constant. Two frequency proportional cases are also nearly the same. A slight difference may come from the difference of definition of damping as described above.

It is noted that considerable difference is seen in the result between constant damping and Rayleigh damping. Therefore, the engineer should recognize Rayleigh damping is not relevant damping from the point of view of accuracy although it helps stability of the numerical integration very much.

Through this example, one can see the effect of high frequency component. The damping characteristics of the Rayleigh damping (stiffness proportional damping) suggest a crisis to analysis the behavior of the ground under both horizontal and vertical input. Generally, predominant frequency of the ground against vertical displacement is much larger than that under horizontal displacement, and predominant frequency of the input motion is much higher than that of horizontal input motion. In other words, interested frequency component is much higher for vertical movement than for horizontal movement.

As discussed in the preceding, in order to obtain a reasonable result, relevant damping is required. Since the value of coefficient is determined focusing on the horizontal behavior, high frequency component is much over damped; high frequency component is not well reproduced. Therefore, if both horizontal and vertical behavior is analyzed at the same time, vertical behavior is not reproduced. Future improvement will be required to avoid this behavior.

8 LIQUEFIED MATERIAL

It seems obvious that liquefied material behaves as if it is liquid by looking at sand boils. However, if sand is loaded in the laboratory, is always has stiffness. In other words, it behaves like solid. There were long discussions whether liquefied sand behaves like solid or liquid (see ref. 20, for example), but it seems to come a conclusion. According to ref. 20, the liquefied sand has very little or nearly zero stiffness, but skeleton structure is maintained. Therefore, additional driving force such as drag force or small vibration after the main shock is necessary to make the soil into liquid state. By these driving force, the soil loose skeleton structure and behaves like liquid. However, if sand is deformed at this state, another skeleton structure will be produced, then sand behaves like solid. This alternate repetition of solid and liquid phases looks as if sand behaves like liquid with large viscosity.

This mechanism indicates that liquid phase material property cannot be obtained in the laboratory by a simple or conventional test. At least, shaking table test or centrifugal test will be required. Back analysis of the observation of liquefaction-induced flow is also a powerful tool, but there is little observation.

It is also known that deformability or viscosity of the sand after liquefaction depends on the loading cycles applied after the onset of earthquake.

Although mechanism seems to be made clear, quantitative evaluation of the property is still not developed. The condition under which phase transform occurs is not made clear, too. In addition, computer codes that can deal with liquefied state are not developed.

Effort should be necessary to find the condition when phase transform from solid to liquid or from liquid to solid occurs. Computer codes that can treat the phase transform will also be required.

9 LIMITATION OF ANALYSIS AND REQUIREMENT FOR FUTURE DYNAMIC RESPONSE ANALYSIS

Figure 16 is the result of the analysis of fill embankment that was damaged during the 1993 Kushiro-oki earthquake[21]. The fill failed as shown in Figure 17; considerable downward displacement occurred and many cracks were seen at the surface. The analysis explains that damage to fill was caused by soil liquefaction at the lower part of the fill, which agrees with observation.

In this sense, we can conclude that the analysis succeeded. However, does it true? As pointed out by the authors, actual failure is slope instability type judging from the observation of failure, which is different from the analysis. In addition, since bottom of the liquefied layer and top of the nonliquefied layer under the liquefied layer have common node, deformed shape may be different from the observation near here.

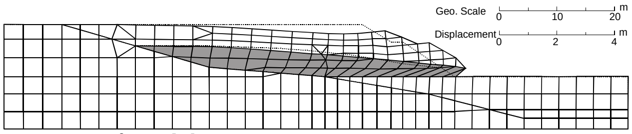


Figure 16 Deformed shape


Figure 17 Damage to fill embankment during the 1991 Kushiro-oki earthquake

One may think that joint element can solve this problem, but it is not. Joint element works when slip surface is linear, but does not work well for curved slip surface. In addition, even if joint element works, one will meet another problem where to place joint element. Placing it at all surface between elements is not realistic solution. It will also be recognized that shape of element will affect the result.

Another problem is appearance of cracks. If crack appears, excess porewater pressure dissipates quickly through them. Therefore, it seems difficult to keep liquefied states. It is also seems difficult to reproduce this kind of behavior by conventional numerical analyses. Moreover, when crack appears, the ground is now discontinuous material whereas finite element analysis assumes continuum material; the fundamental assumption of the effective stress analysis does not hold.

Considering this example and discussion in the previous section, applicable range of current dynamic response analysis seems within the onset of failure or just before the start of large deformation. Soil behaves like liquid after the onset of liquefaction, and cracks breaks fundamental assumption of the finite element analysis. Another approach will be required to express the behavior after failure.

It is noted, however, that, as discussed in the introduction, requirement on the dynamic response analysis does not stay before the failure, but expanding to the behavior after the failure. At present, simplified static method such as ALID[22] and the method proposed by Towhata[23] have been used after the liquefaction, and Newmark type analysis, in which displacement is obtained by integrating acceleration while sliding occurs, or method to focus on residual displacement[24] have been used in the case that liquefaction does not occur. It is obvious that these methods are theoretically less accurate because only limited factors are taken into account, and the process to failure is not taken into account. A continuous analytical method will be required in future.

10 CONCLUDING REMARKS

In this paper, the author intends to show where errors will be produced at many stages required for the dynamic response analysis from developing the governing equations to the actual calculation. Through the examples introduced in this paper, the author intends to show considerable amount of error may be produced on various stages, among which there are cases that the engineer have been believed to be common sense.

As discussed in the introduction of this paper, total error of the dynamic response analysis is some kind of sum of the error in each stage. Since final error cannot become smaller than the largest error among the stages, it is the most important to reduce it. At the same time, it is also important to reduce the error at each stage although drastic improvement may not be seen by reducing the error in only one stage.

Following researches and efforts will be especially required in order to respond various requirements for the output of the dynamic response analysis, and to reduce error of the dynamic response analysis.

1) A new test method should be developed to obtain the behavior of soil at large strain; conventional test methods, i.e., dynamic deformation characteristics test and liquefaction strength test give little and limited information.

2) Since it is well known that soil sample is disturbed by sampling, development of in-situ test method will be required in order to grasp the in-situ characteristics of soil. It is also important to obtain initial stress state.

3) Development of the analytical procedure at large strains and after the failure will be required.

4) It is impossible to represent the nature of the dynamic response by one index, but a group of indices should be found to evaluate the accuracy of the dynamic response analysis.

5) More vertical array earthquake observations will be required to investigate accuracy of the dynamic response analysis.

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