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A COUPLED BOUNDARY AND FINITE ELEMENT METHOD FOR THE TIME MARCHING ANALYSIS

Terumi TOUHEI[1] and Nozomu YOSHIDA[2]

[1]Nuclear Engineering Division, Sato Kogyo Co. LTD.,
Chuo-ku, Tokyo, Japan
[2]Engineering Research Center, Sato Kogyo Co. LTD.,
Atsugi, Japan

SUMMARY

Calculations of ground motion during the earthquake by the use of coupled finite element and boundary element method is presented. The procedure for time marching analysis is similar to the Newmark's beta method; the same time increment can be used in both the finite element and boundary element regions. An alluvial valley is analysed and clear feature of the behavior is obtained.

INTRODUCTION

Lately, boundary element method or coupled boundary and finite element method has come to be used in the dynamic response analysis of ground. In almost all of these analyses, the boundary element region is formulated in frequency domain. The reason seems to be the complicated formulation as well as the difficulty to obtain a stable solution in time marching analysis. However, it is necessary to formulate them in the time domain to analyze transient response and/or nonlinear behavior because accurate result cannot be obtained by the analysis in frequency domain in these cases.

Coupled finite element and boundary element method in time domain were investigated by Fukui et al and by the authors. Fukui et al solved finite element region and boundary element region separately by using different time increment and employed iterative procedure and least square method to combine them, which comes from the requirement to the time increment to obtain stable solution in each region. The authors employed the method of weighted residuals to combine finite element region and boundary element region. This method have advantages that the same time increment can be used in both finite element and boundary element regions and no iterative procedure is required, hence can save more computing time, and through the numerical examples, it was shown that more accurate response can be obtained by the proposed method even under the complicated input wave. This paper presents the result of analysis of an alluvial valley by the use of this method.

METHOD OF ANALYSIS

<u>Finite Element Formulation</u> Finite element formulation of the wave propagation equation for the scalar wave motion (SH wave) is obtained by the conventional techniques as

$$[M]\{\ddot{u}\} + [K]\{u\} = \{P(t)\} \qquad (1)$$

where [M] and [K] indicate mass and stiffness matrices, respectively, $\{\ddot{u}\}$ and $\{u\}$ are acceleration and displacement vectors, respectively, and $\{P(t)\}$ indicates external force vector. When there is no body force except inertia force, nonzero component of $\{P(t)\}$ appears at the freedom on the interface between the finite element region and boundary element region.

Boundary Element Formulation The boundary integral equation for the scattering SH wave in the region which has boundary Γ as shown in Fig. 1 under the condition that there is no scattering wave at time t=0 is expressed as

$$\varepsilon u(r,t) + \int_0^t \int_\Gamma T(r,t:r',t')u(r',t')dr'dt' = \int_0^t \int_\Gamma G(r,t:r',t')\sigma(r',t')dr'dt' + \bar{u}(r,t) \tag{2}$$

where ε is a constant determined from the shape of the boundary, $\bar{u}$ denotes incident wave, σ denotes traction, t indicates time, and G and T are Green's functions correspond to the displacement and traction. By dividing the boundary Γ into boundary elements and by dividing the time axis into N increments, Eq. 2 is descretized as

$$\sum_{k=1}^{N} [H^{NK}]\{u^k\} = \sum_{k=1}^{N} [G^{NK}]\{\sigma^k\} + \{\bar{u}^N\} \tag{3}$$

Here $\{u^k\}$ and $\{\sigma^k\}$ indicate displacement and traction vectors at k^{th} time step, respectively, and $[H^{NK}]$ and $[G^{NK}]$ are coefficient matrices of influence correspond to them, respectively.

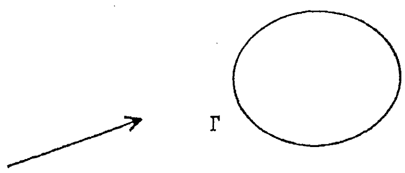

Fig. 1 Analysed Model by Boundary Element Method

By multiplying the distribution matrix [D] into Eq. 3 and by rearranging it, the nodal force P^N versus nodal displacement u^N relation at N^{th} time step is obtained as follows:

$$\{P^N\} = [K^*]\{u^N\} - [D]\{f^N\} \tag{4}$$

Here, $[K^*] = [D][G^{NN}]^{-1}[H^{NN}]$, and $\{f^N\}$ is an external load term determined from the tractions and displacements before the N^{th} time step and incident wave at N^{th} time step. Equation 4 is the governing equation in the boundary element region. It is noted that Eq. 4 can be interpreted as the discretized expression of the following equation which is obtained by applying the deconvolution into Eq. 2:

$$\sigma(r,t) = \int_0^t \int_\Gamma \lambda(r,t:r',t')u(r',t') - \hat{f}(r,t) \tag{5}$$

where λ is an integral operator obtained from Green's function, and $\hat{f}(r,t)$ is external load term derived from incident wave and Green's function.

Coupling the Finite and Boundary Element Regions It is necessary to satisfy the kinematic condition and equilibrium condition on interface between the finite element and boundary element regions. The following governing equation is obtained by applying these conditions into Eqs. 1 and 4:

$$[M]\{\ddot{u}\} + [K]\{u\} + \int_0^t [\Lambda(t:t')]\{u(t')\}dt' = \{f(t)\} \tag{6}$$

where $[\Lambda(t:t')]$ is a matrix obtained by discretizing Eq. 5 in space, and $\{f(t)\}$ is external load term vector obtained when replacing Eq. 5 into the relation between the nodal force and displacement. As shown, the coupled equation are integral-differential equation with respect to time. The method of weighted residuals is employed to relax the equilibrium condition so as to solve this equation. The relaxed equilibrium condition is expressed by the use of weight function $w(\xi)$ as

$$\int_{-1}^{+1} w(\xi)(\{P\}_F + \{P\}_B)d\xi = \{0\} \tag{7}$$

where $\{P\}$ indicates nodal force and subscripts F and B indicate finite element and boundary element regions, respectively. The variable ξ is a dimensionless time expressed as

$$\xi = \frac{t}{\Delta t} - N + 1 \tag{8}$$

The final governing equation is obtained as follows by integrating Eq. 7 using second order displacement functions in both finite element and boundary element regions, 0^{th} order functions as interpolation function of traction for time on the boundary of boundary element region, and using the even function for $w(\xi)$, as

$$\begin{aligned}[M + \beta\Delta t^2K + \Delta t^2K^*/2]\{u^N\} &= [2M - (1-2\beta)\Delta t^2K - \Delta t^2K^*/2]\{u^{N-1}\} \\ &+ [-M - \beta\Delta t^2K]\{u^{N-2}\} + \Delta t^2[D](\{f^N\} + \{f^{N-1}\})/2 \end{aligned} \tag{9}$$

The same time increment is used in both boundary element and finite element regions to integrate the governing equation by this method, which procedure is similar to the Newmark's beta method except that there exist boundary element term. The solutions of Eq. 9 gives displacement response of the ground. Velocities and acceleration responses are obtained by differentiating the interpolation function of displacements for time, which is employed when discretizing Eq. 2 into Eq. 3, with respect to time.

DYNAMIC BEHAVIOR OF ALLUVIAL VALLEY

Analysis The dynamic behavior of an alluvial valley is investigated. The model is soft soil on the rock base and is shown in Fig. 2. The incident SH wave enters the valley from left hand side at the incident angle of 45 degree from vertical axis. In the analysis, six nodes triangular element is used as the finite element, second order function is used to interpolate the displacement of the boundary element, which interpolation function is the same order function with the displacement function of finite elements, and first order function is used to interpolate tractions in space. The Green's function for the semi-infinite region is used in the analysis of the boundary element region.

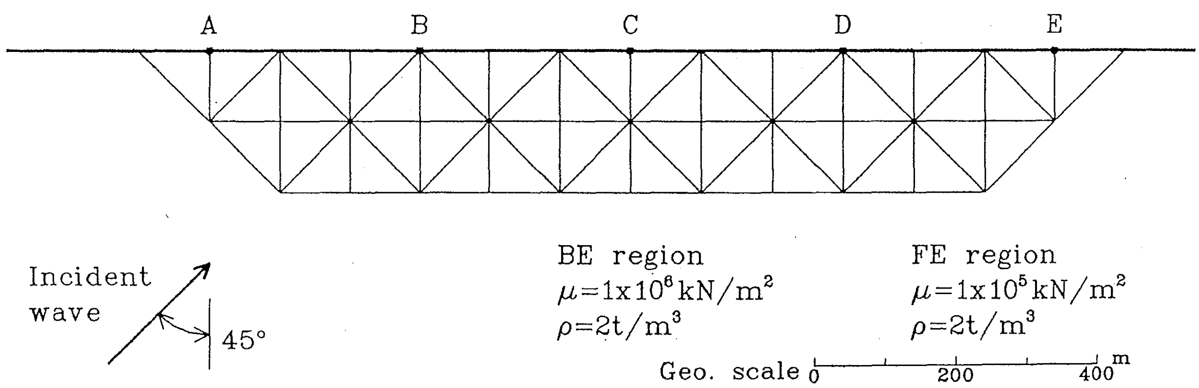

Fig. 2 Alluvial Valley Model, Finite Element Mesh and Material Properties (μ: shear modulus, ρ: mass density)

The El-Centro 1940 wave is used as the incident SH wave whose time increment is 0.01 second, duration is 10 seconds and maximum acceleration is normalized to 0.13G, which is shown in Fig. 3. The displacement time history of the incident wave, which is required to compute the right hand term of Eq. 9, is computed using the FFT technique from the acceleration time history, which displacement is shown in part(b) of Fig. 3. Since time increment to be used in the analysis so as to obtain accurate response relates the shear wave velocity of the boundary element region and the length of boundary elements, the time increment are determined 0.05 second to solve Eq. 9. Therefore high frequency component of the wave may be filtered in the analysis. Here it is noted that original time increment, 0.01 second, is used to compute external load term {f} in Eq. 9. Smaller time increment can be used if the length of the boundary element is shorter (minimum length of boundary element is 100m in the model in Fig. 2). The value of 1/4 is used for β.

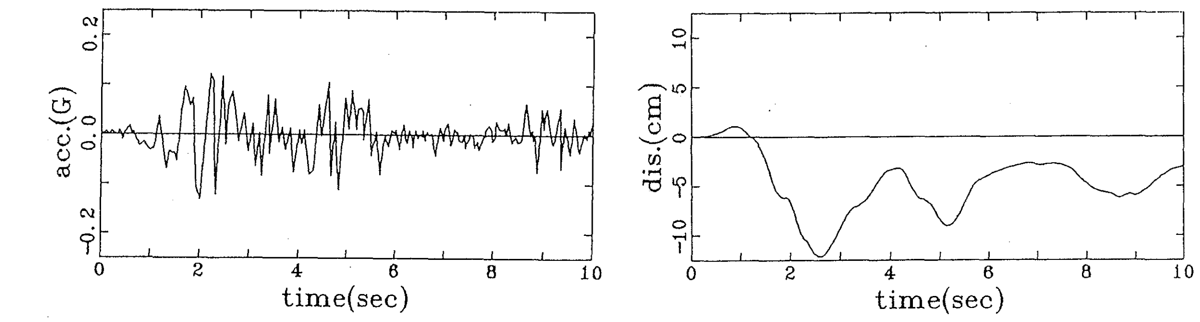

Fig. 3 Acceleration and Displacement Time Histories of Incident Wave

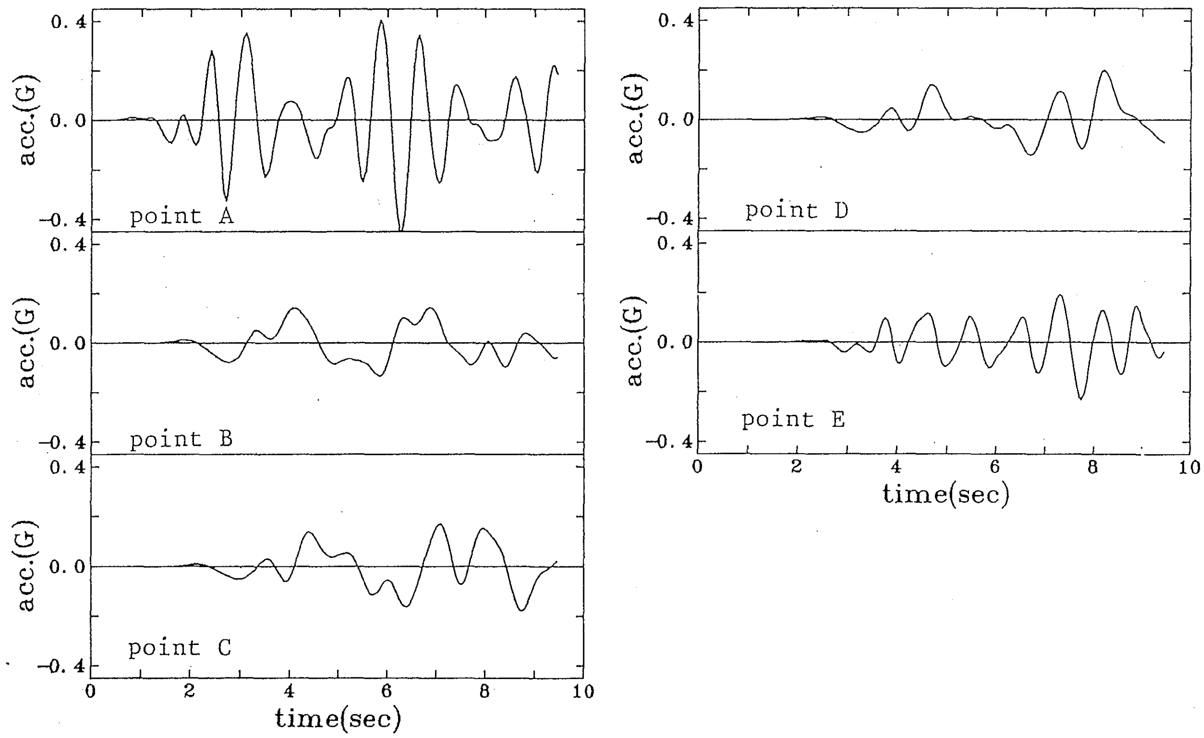

Fig. 4 Acceleration Response at the Surface

Characteristics of ground vibration at alluvial valley   The acceleration time histories at points A, B, C, D and E in Fig. 3 are shown in Figs. 4. The maximum acceleration at point A are larger than those at the other points, which indicate that the energy of earthquake wave concentrate near point A as the wave reflects and refracts in the complicated manner near the dipping interface.

The predominant period of the response acceleration at points B, C and D seems longer than those at points A and E which locate close to the dipping interface. The depth at points A and E are smaller than that at points B, C and D, which difference appears in the predominant period of the response.

The predominant period of the acceleration response is much larger than that of incident wave. Moreover, the earlier part of the waveform at points B, C and D are similar to each other and obvious phase lag is observed. These phenomena indicate that surface wave is generated in the alluvial valley. To see it more clearly, the displacement shapes at the surface at every time step are shown in Fig. 5. The phase velocity of the wave which travells from left to right hand side is about 500 m/second which is a half of the phase velocity of SH wave in the horizontal direction, 1000m/second. Figure 6 shown dispersion property of fundamental mode of Love wave at the horizontally layered part of the alluvial valley. The period of the Airy phase of this wave is about 4 seconds and the phase velocity is about 500 m/second, which agrees with the calculated wave very much. Therefore the existence of Love wave is confirmed. Moreover, it is recognized that the surface wave have much effect on the dynamic response of the alluvial valley.

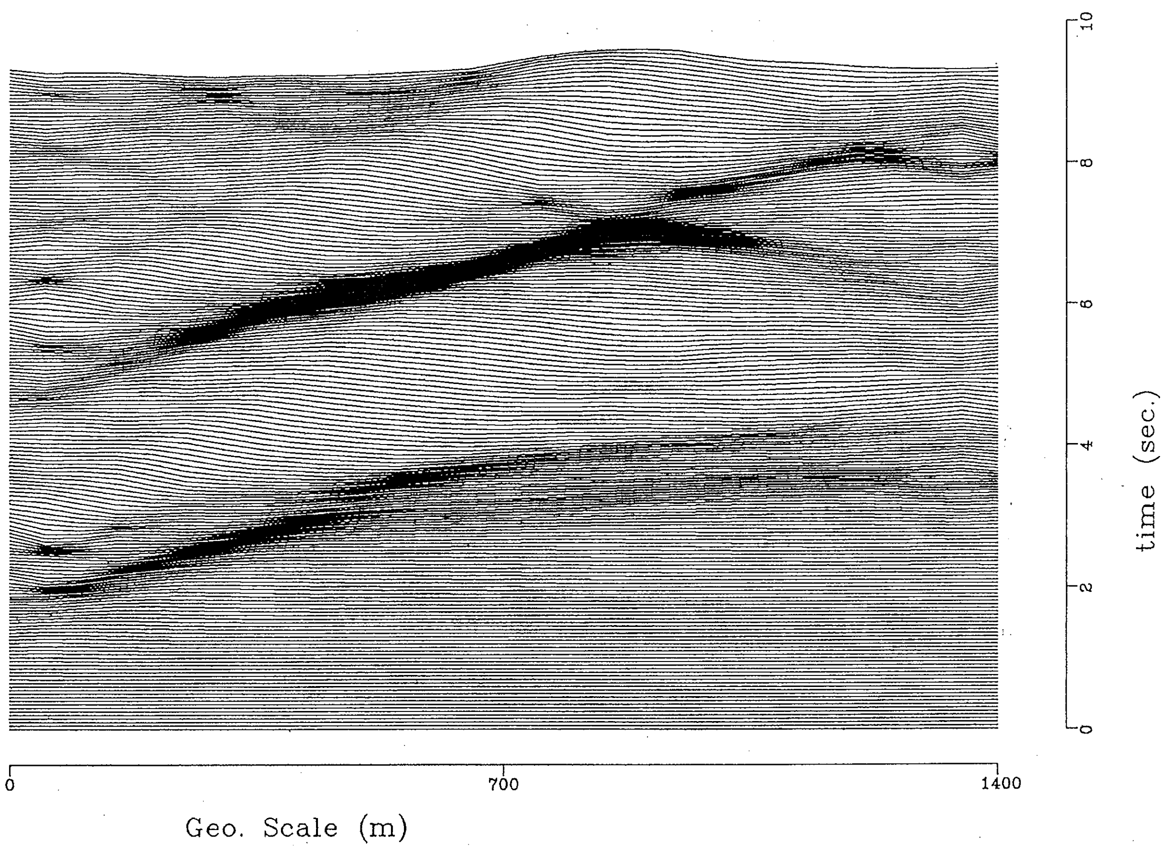

Fig. 5 Displacement Shapes at the Surface

Although the early part of the waveform at points B, C and D are similar to each other, the latter part of them cannot be said to be similar. This indicate that the waves repeatedly reflected at the soft soil-rock base interface and the surface come to affect the response, which is also clearly seen in Fig. 5. It is also noted that the acceleration of the incident wave becomes maximum in an early time but that of the response at the surface becomes maximum in the late time, therefore the effect of reflected wave seems to become predominant.

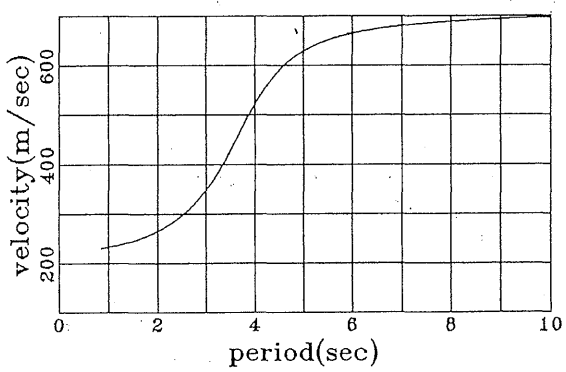

Fig. 6 Dispersion Property of Love Wave at Horizontally Layered Part

CONCLUDING REMARKS

The dynamic response of the soft ground above the rock base is investigated by the use of coupled finite element and boundary element method in time domain. Since this method directly computes the solution in time domain, it is useful, for example, in the transient wave analysis and nonlinear analysis. The numerical calculation of the ground gives clear feature of the dynamic response at an alluvial valley The surface wave is generated and have predominant effect of the behavior. Moreover, the reflected waves also have predominant effect as time goes on.

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