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Stress Dilatancy Relationship of Sand under Cyclic loading.

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ABSTRACT

Stress dilatancy behavior of sand under cyclic loading is examined with special reference to the non-coaxiality between principal stress and principal strain direction. The stress dilatancy relationship is derived from normalized shear work.

1. INTRODUCTION

Stress dilatancy relationship for monotonic loading conditions has been investigated by many researchers, whereas the same under cyclic loading has been investigated to a very limited extent. Among the several relationships derived by many researchers, the stress dilatancy relationship based on normalized shear work is examined. Therefore, the incorporation of noncoaxiality as mentioned above is inevitable to maintain the physical meaning of the shear work when dealing with the stress and stain invariants.

The modification proposed for the stress-dilatancy relationship here is observed through drained Torsional simple shear test data (Pradhan et. al 1989)

2. TESTING METHOD & MATERIAL

Torsional simple shear test data (by Prdhan et al. 1989) on Toyoura sand is used. Relative density of the sample tested is 75%.

3. STRESS & STRAIN PARAMETERS

1. Stress invariants

$$p = \frac{1}{2}(\sigma_x + \sigma_y) \qquad .....(1)$$

$$q = \sqrt{\frac{(\sigma_y - \sigma_x)^2}{4} + \sigma_{xy}^2} \qquad .....(2)$$

The angle formed by the major principal stress σ_1 with the vertical or y-axis is given by

$$\tan 2\beta_\sigma = \frac{2\sigma_{xy}}{\sigma_y - \sigma_x} \qquad .....(3)$$

2. Strain invariants

$$d\varepsilon_v = d\varepsilon_x + d\varepsilon_y = d\varepsilon_1 + d\varepsilon_3 \qquad .....(4)$$

$$d\bar{\varepsilon} = \sqrt{(d\varepsilon_y - d\varepsilon_x)^2 + 4d\varepsilon_{xy}^2} \qquad .....(5)$$

The angle $\beta_{d\varepsilon}$ between the major principal strain increment $d\varepsilon_1{}^p$ and the vertical or y-axis is defined by

$$\tan 2\beta_{d\varepsilon} = \frac{2d\varepsilon_{xy}^p}{d\varepsilon_y^p - d\varepsilon_x^p} \qquad .....(6)$$

4. STRESS DILATANCY RELATIONSHIP

Consider the equation of energy dissipation which is used as the primary form of stress dilatancy relation employed in this study:

$$\begin{aligned} dW^p &= \sigma_{ij} d\varepsilon_{ij}^p \\ &= \sigma_{11} d\varepsilon_{11}^p + \sigma_{22} d\varepsilon_{22}^p + \sigma_{33} d\varepsilon_{33}^p + \\ &\quad 2\sigma_{12} d\varepsilon_{12}^p + 2\sigma_{23} d\varepsilon_{23}^p + 2\sigma_{31} d\varepsilon_{31}^p \end{aligned} \qquad .....(7)$$

Since the tensors σ_{ij} and $d\varepsilon_{ij}$ share the same axes, the above expression is always valid. However, the equation of energy dissipation written in terms of principal tensors given by

$$dW^p = \sigma_1 d\varepsilon_1^p + \sigma_2 d\varepsilon_2^p + \sigma_3 d\varepsilon_3^p \qquad ....(8)$$

may not be always valid as the principal stress tensor and principal strain increment tensor may not always satisfy coaxiality. Therefore, the equation of energy dissipation should be calculated based on Eqn. 7. This equation, however is not very convenient due to six dimensional form. In the following , this equation is rewritten in terms of stress and strain invariants is rewritten in terms of stress and strain invariants while non-coaxiality is taken into account (Gutierrez, 1989).

For the clarity, two dimensional form is considered hereafter.

$$dW^p = \sigma_{ij} d\varepsilon_{ij}^p$$
$$= \sigma_x d\varepsilon_x^p + \sigma_y d\varepsilon_y^p + 2\sigma_{xy} d\varepsilon_{xy}^p \quad .....(9)$$

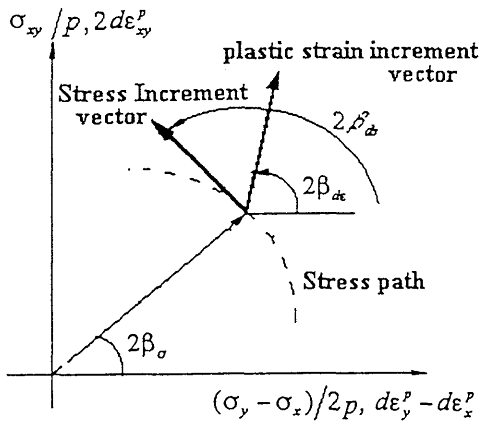

Figure 1: Graphical presentation of stress and strain increment direction

The stress components can be written in terms of invariants p and q and angle β_σ i.e.,

$$\sigma_x = p - q\cos 2\beta_\sigma \quad .....(10a)$$
$$\sigma_y = p + q\cos 2\beta_\sigma \quad .....(10b)$$
$$\sigma_{xy} = q\sin 2\beta_\sigma \quad .....(10c)$$

Similarly, the plastic strain increment components $d\varepsilon_{ij}^p$ can be written in terms of the invariants $d\varepsilon_v^p$ and $d\varepsilon^p$ as

$$d\varepsilon_x^p = \tfrac{1}{2} d\varepsilon_v^p - \tfrac{1}{2} d\bar{\varepsilon}^p \cos 2\beta_{d\varepsilon} \quad .....(11a)$$
$$d\varepsilon_y^p = \tfrac{1}{2} d\varepsilon_v^p + \tfrac{1}{2} d\bar{\varepsilon}^p \cos 2\beta_{d\varepsilon} \quad .....(11b)$$
$$d\varepsilon_{xy}^p = \tfrac{1}{2} d\bar{\varepsilon}^p \sin 2\beta_{d\varepsilon} \quad .....(11c)$$

Substituting Eqns 10 and 11 in Eqn 9 yields

$$dW^p = p\,d\varepsilon_{vd}^p + q\,d\bar{\varepsilon}^p \cos(2\beta_\sigma - 2\beta_{d\varepsilon})$$
$$dW^p = p\,d\varepsilon_{vd}^p + q\,d\bar{\varepsilon}^p \cos 2\psi \quad .....(12)$$

Where β_σ and $\beta_{d\varepsilon}$ are principal stress direction and the direction of principal strain increment. Therefore, $\psi = (\beta_\sigma - \beta_{d\varepsilon})$ is the angle of non-coaxiality.

At present, the stress dilatancy relationship is derived assuming that the normalized energy as a function of plastic shear strain, ε^p

$$d\Omega^p = \frac{dW^p}{p} = d\varepsilon_{vd}^p + \frac{q}{p} d\bar{\varepsilon}^p \cos 2\psi = \mu d\bar{\varepsilon}^p$$

Yielding

$$\frac{d\varepsilon_{vd}^p}{d\bar{\varepsilon}^p} = \mu - c.\frac{q}{p} \quad .....(13)$$

Where $c = \cos 2\psi$

5. EXPERIMENTAL OBSERVATIONS

The Torsional simple shear data from Pradhan et. al. (1989) is used.

As mentioned above the stress-dilatancy relationship is derived from normalized shear work. Fig. 2 shows the relationship between normalized shear work and the accumulated plastic shear strain ε^p.

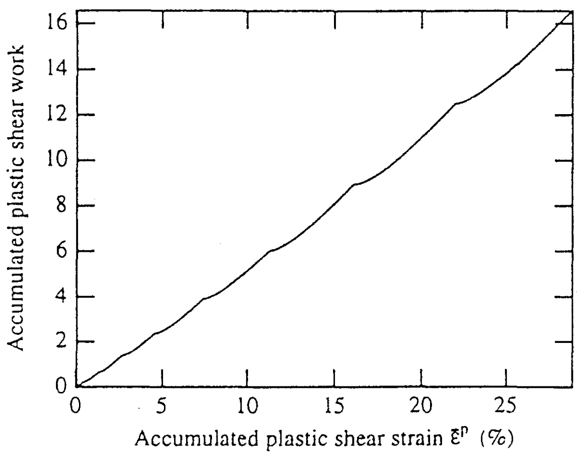

Figure 2: Relationship between plastic shear work and plastic shear strain.

Although, the shear work accumulated positively irrespective of loading or unloading in terms of q/p (see Fig. 3), it is interesting to note that c.q/p produce loading path during a complete cycle (see Fig. 4). Therefore, the use of c.q/p may give us avenue for better understanding/interpretation. Hence, c.q/p is employed hereafter.

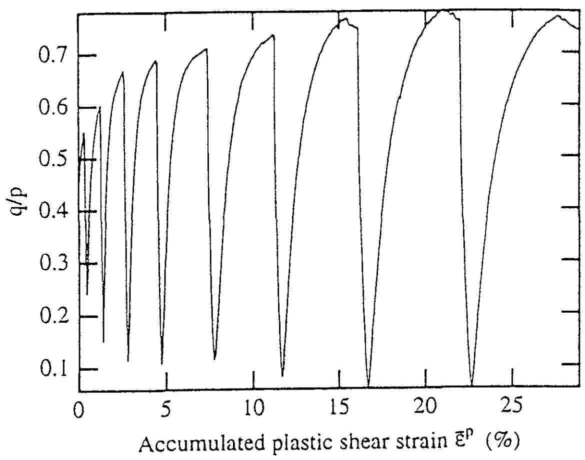

Figure 3: Relationship of shear stress ration q/p versus plastic shear strain

Further, It can be observed from Fig. 2 that the rate of accumulation of shear work is retarded at the beginning of each loading steps. However, in most of the constitutive models linear relationship is assumed between normalized shear work and accumulated plastic shear strain, thus assuming

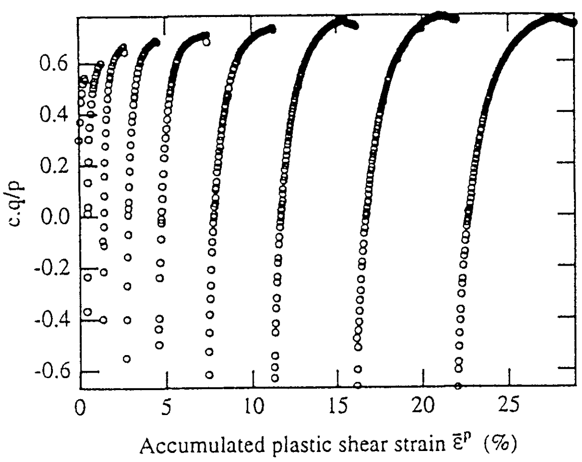

Figure 4: Relationship of c.q/p versus plastic shear strain

constant value for μ as given in Eqn. 13. Some researchers proposed μ to be vary with cumulatively increasing parameter.

In order to examine the variation of μ, the value of μ is exclusively calculated using Eqn. 13 for the available test data. Fig. 5 illustrate the variation of μ with c.q/p. It can be observed that the μ increases with the increase of c.q/p within the cycle and reaches a value which is related to phase transformation angle (ϕ_c) of the material.

6. A MODIFICATION

As observed in Fig. 5, following variation of μ against c.q/p is assumed.

$$\mu = \sin\phi_c \left[\frac{c\dfrac{q}{p} - \left(c\dfrac{q}{p}\right)_i}{\sin\phi_c - \left(c\dfrac{q}{p}\right)_i} \right] \qquad \text{if } c\frac{q}{p} \le \sin\phi_c$$

$$\mu = \sin\phi_c \qquad \text{if } c\frac{q}{p} \ge \sin\phi_c \qquad .....(14)$$

Where $\left(c.\dfrac{q}{p}\right)_i$ is the value of c.q/p at the beginning of each loading step.

Constant $\mu=\sin\phi_c$ is assumed during virgin and/or monotonic loading.

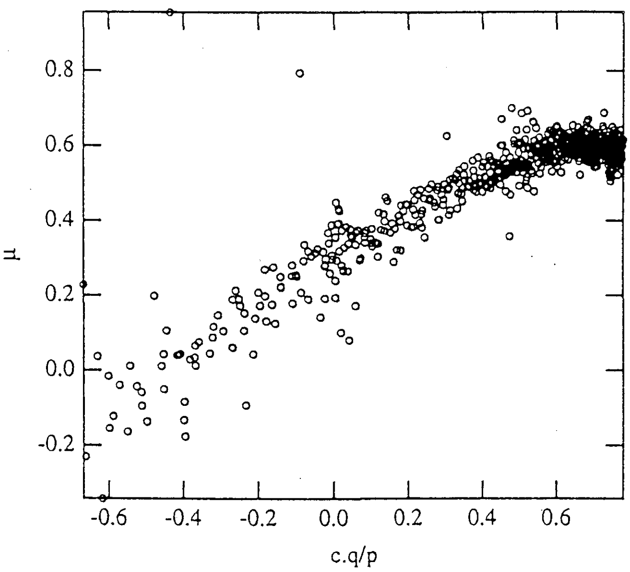

Figure 5: Variation of parameter μ against c.q/p

In order to compare the proposed modification, only the volumetric strain is calculated using the stress-strain relation from the experimental data.

Therefore, it is necessary to mention here that the calculated volumetric strain is not solely based on a complete constitutive model, but only by the strain-dilatancy relationship in which volumetric strain increment is assumed to be the only unknown.

Fig. 6 compares the calculated and experimental volumetric strain due to shearing of the soil sample.

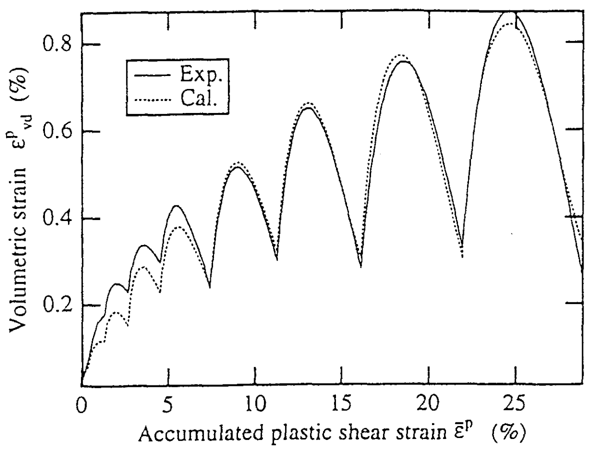

Figure 6: Plastic volumetric strain due to shearing

Fig. 7 and Fig. 8 illustrate the standard plot of stress-dilatancy relation in terms of stress ratio (c.q/p) versus strain ratio ($d\varepsilon_{vd}/d\varepsilon^p$) for experimental and calculated data respectively.

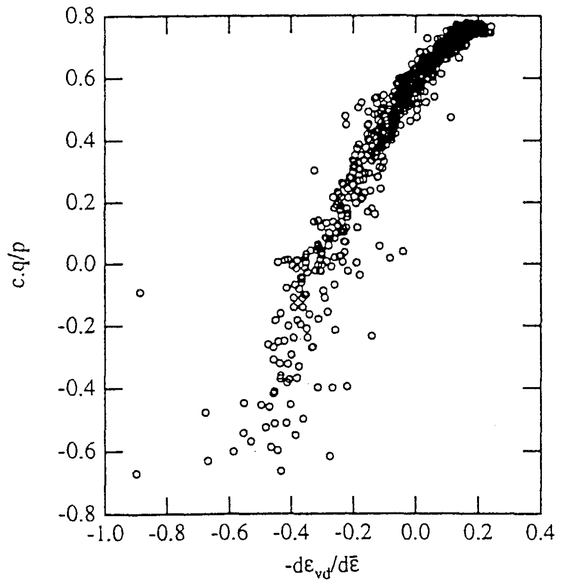

Figure 7: Stress-Dilatancy relationship based on experimental data

7. CONCLUSION

From the available test data it can be observed that μ should be varied within the cycle in quantitative prediction of volumetric strain due to shearing .

The data available is not sufficient to verify the validity of variation of μ, thus it should be checked for varying test conditions.

The variation of μ may be affected very much by the over consolidation ration and the change in b-value during the test.

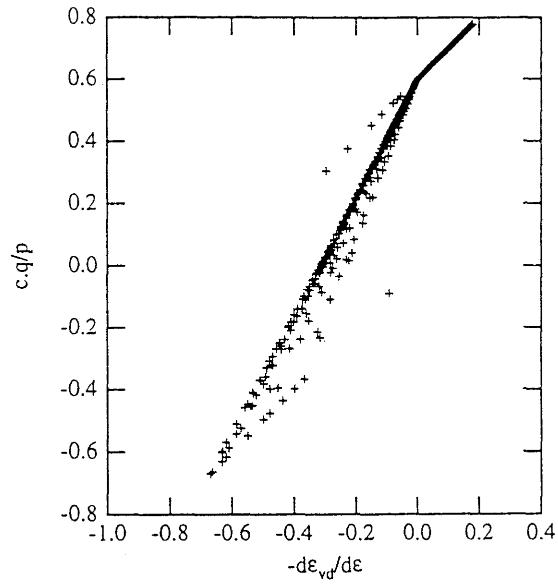

Figure 8: Stress-Dilatancy relationship based on assumed variation of μ

8. REFERENCE

1. Pradhan T.B.S., Tatsuoka F. and Sato Y., (1989), Experimental Stress-Dilatancy Relations of Sand Subjected to Cyclic Loading, Soils and Foundations, Vol. 29, No. 1, pp 45-64.

2. Gutierrez M., (1989), Behavior of Sand During Rotation of Principal Stress Directions, D.Eng. Thesis, University of Tokyo.