# Dilatancy characteristics of sand

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## Abstract

Dilatancy characteristics of sand under cyclic loading including cyclic mobility behavior and behavior at large strains after liquefaction is investigated. Series of torsional shear test data and torsional simple shear test data, including both monotonic and cyclic loading, have been processed. First, stress-dilatancy relationship is developed using drained test results based on normalized shear work. Volumetric strain characteristics in wide range of confining pressure including low confining pressure levels are then investigated using undrained test data and stress-dilatancy model established using drained test data. It is shown that volumetric strain characteristics are similar to the one after liquefaction, except that the range of low stiffness region is smaller than that. This indicates that conventional constitutive model may not be able to simulate the behavior of sand at low confining pressure. It is also recognized that material properties deteriorate gradually with the application of cyclic load.

## 1 Introduction

Dilatancy characteristics play an important role in the liquefaction behavior of sand. Many researches have been conducted to develop stress-strain models (Dafalias, 1994; Towhata, 1989). Yoshida (1991) pointed out that many constitutive models based on the plasticity theory have its own difficulties in expressing the behavior of sand during and after the liquefaction. One is that, in the model, hysteritic behavior becomes stable fairly quickly after phase transformation whereas in the laboratory test, shear strain increases with loading. The other is that volume change due to dissipation of excess pore water pressure derived from models is nearly constant and much smaller than that of test in which volume change depends on the loading applied after liquefaction (Nagase and Ishihara, (1988)).

Development in this field comes from the researches focused on the behavior after liquefaction including liquefaction-induced large permanent displacement. Yasuda et al. (1994) conducted laboratory test in which sand is subjected to monotonically increasing strain after liquefaction. Yoshida et al. (1994a) processed this test data and found the existence of an unstable region near the liquefaction, the extent of this region depends on the loading cycles. Since their investigation focused on the behavior after liquefaction, transient phenomena at liquefaction was not clear.

In this paper, we investigate the dilatancy behavior in transient region at liquefaction based on the cyclic load test result and show that it can be expressed in a continuous way from transient to the behavior after liquefaction.

## 2 Behavior after liquefaction

Yoshida et al. (1994b) measured volume change due to change in effective confining pressure, in the process of excess pore water pressure dissipation after liquefaction, as shown in Fig.1. They pointed out that large volume change occurs just at the beginning of drainage, and its amount depends on the amount of loading applied after liquefaction. They also pointed out that in the conventional expression of bulk modules in which bulk modules is proportional to some power of effective confining pressure can express the behavior at higher effective confining pressure, but cannot express the behavior at low effective confining pressure close to zero. Yoshida and Finn (1994) proposed an equation expressing this behavior in the form,

$$\frac{p'}{p'_0} = \frac{e^{\frac{\varepsilon_v}{c}} - 1}{e^{\frac{\varepsilon_{v0}}{c}} - 1} \tag{1}$$

where c is a constant and $\varepsilon_{v0}$ is volumetric strain at reference mean stress. As shown in Fig. 1, agreement between test result and Eq. 1 is good.

Yasuda et al. (1994) conducted laboratory test of sand in which cyclic shear load is applied first even after the liquefaction and then shear strain is increased monotonically. Figure 2 shows an example of test result. They pointed out that there appears a low stiffness region at the beginning of monotonic loading. FL (Safety factor against liquefaction) value is used as an index of loading after liquefaction; FL value decreases as the amount of loading after liquefaction increases. The range of low stiffness region depends on FL value as seen in Fig.2. Yoshida et al. (1994a) analyzed this test and showed that the behavior can be simulated if shear modules at small strain also has the shape given by Eq. 1. Their analysis is shown in Fig. 2 as well. The agreement between the test and the analysis is good. They also pointed out that various material properties such as internal friction angle change with the amount of load applied after liquefaction.

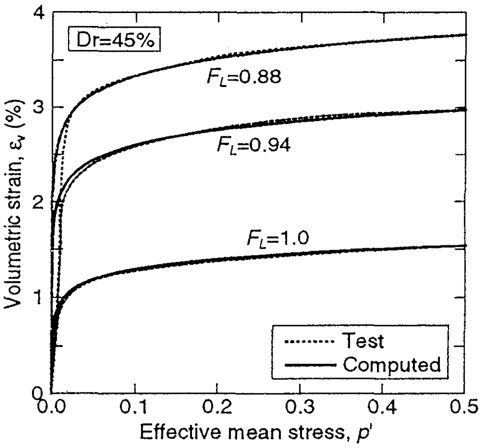


Figure 1: Volumetric strain characteristics during the excess pore water pressure dissipation after liquefaction.

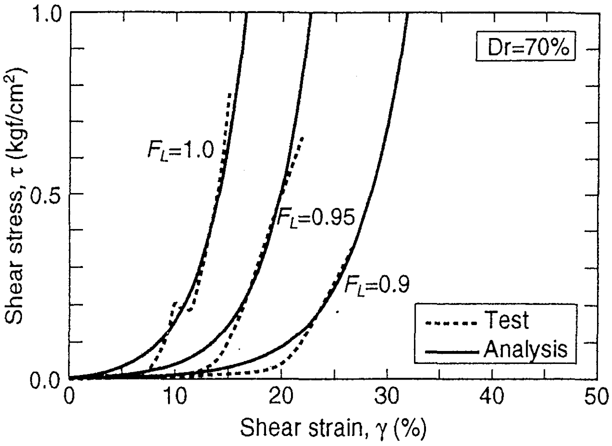


Figure 2: Stress-strain relationship of sand subjected to monotonic loading after lique faction.

In summary, after the occurrence of liquefaction, material property changes, because structure of soil skeleton is disturbed very much due to the occurrence of liquefaction at which inter granular force is very small. At low confining pressure, a new structure is not well developed, therefore stiffness is very small. When confining pressure increases, new structure is formed. Conventional theory seemed to be based on latter stage where material behavior is somewhat stable.

## 3 Behavior during cyclic loading

### 3.1 Stress & Strain parameters

$$p = (\sigma'_a + \sigma'_t)/2 \quad ; \quad d\varepsilon_v = d\varepsilon_a + d\varepsilon_t$$

$$q = \sqrt{[(\sigma'_a - \sigma'_t)/2]^2 + \sigma'^2_{at}} \quad ; \quad d\bar{\varepsilon} = \sqrt{(d\varepsilon_a - d\varepsilon_t)^2 + (2d\varepsilon_{at})^2}$$

$$\tan 2\beta_\sigma = 2\sigma'_{at}/(\sigma'_a - \sigma'_t) \quad ; \quad \tan 2\beta_{d\varepsilon} = 2d\varepsilon^p_{at}/(d\varepsilon^p_a - d\varepsilon^p_t) \qquad (2)$$

where $\beta_\sigma$ and $\beta_{d\varepsilon}$ are the directions made by principal stress and principal strain increment direction with vertical respectively.

### 3.2 Stress-Dilatancy relationship

Consider the plastic shear work which is used as the primary form of stress dilatancy relation;

$$dW^p = \sigma_{ij} d\varepsilon^p_{ij}$$

In two dimensional form;

$$dW^p = \sigma_{ij} d\varepsilon^p_{ij} = \sigma_a d\varepsilon^p_a + \sigma_t d\varepsilon^p_t + 2\sigma_{at} d\varepsilon^p_{at} \qquad (3)$$

In terms of stress and strain invariant as given in Eq. 2

$$dW^p = p d\varepsilon^p_{vd} + q d\bar{\varepsilon}^p \cos(2\beta_\sigma - 2\beta_{d\varepsilon}) = p d\varepsilon^p_{vd} + q d\bar{\varepsilon}^p \cos 2\psi \qquad (4)$$

where $\psi = \beta_\sigma - \beta_{d\varepsilon}$ is the angle of non-coaxiality; the angle by which the principal stress direction and principal strain direction vary (Gutierrez, 1989).

$\varepsilon_{vd}$ is the volumetric strain due to shearing. Superscript 'p' stand for plastic component of the corresponding strain.

The stress dilatancy relationship is derived assuming that the normalized shear work as a function of accumulated plastic shear strain,

$$d\Omega^p = dW^p / p = d\varepsilon_{vd}^p + (q/p)d\bar{\varepsilon}^p \cos 2\psi = \mu d\bar{\varepsilon}^p$$

Yielding,

$$d\varepsilon_{vd}^p / d\bar{\varepsilon}^p = \mu - c.q/p \qquad (5)$$

where c=cos2$\psi$ , $\mu$(=$\mu_c$; sine of phase transformation angle) is assumed to be a constant

**3.3 Drained condition.**

Peiris et al. (1994) has shown that the dilatancy relation given by Eq.5 must be modified in accurate estimate of volumetric strain development. Modification for the stress-dilatancy relationship is mainly by assuming a varying m, given as follows.

$$\mu = \mu_c \left[ \frac{c\cdot q/p - (c\cdot q/p)_i}{\mu_c - (c\cdot q/p)_i} \right] \qquad \text{if} \quad c\cdot q/p \le \mu_c$$

$$\mu = \mu_c \qquad \text{if} \quad c\cdot q/p \ge \mu_c \qquad (6)$$

Where $(c\cdot q/p)_i$ is the value of $(c\cdot q/p)$ at the beginning of each loading step. Constant $\mu$(=$\mu_c$) is assumed during virgin and/or monotonic loading.

Torsional simple shear test data (by Pradhan et. al (1989)) is re-examined in assessing the validity of the modified dilatancy relationship. Fig.3 illustrates the stress-strain relationship in which loading and unloading paths can be identified.

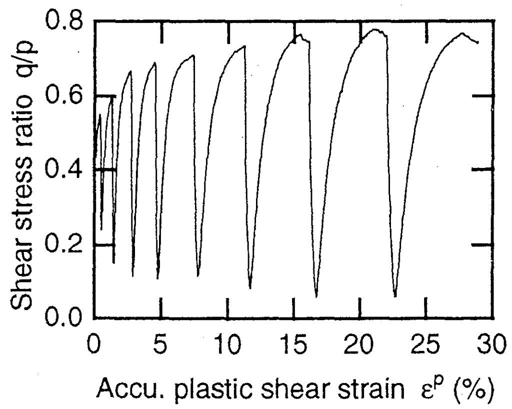


Figure 3: Relationship of Shear stress ratio q/p versus accumulated plastic Shear strain

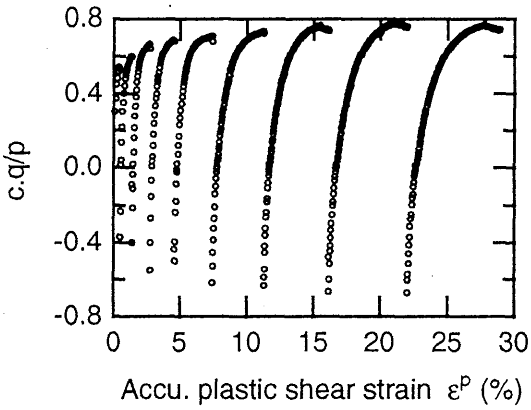


Figure 4: Relationship of Shear stress ratio $c\cdot q/p$ versus accumulated plastic Shear strain *(Loading paths)*

If the non-coaxiality of principle strain increment direction and principle stress direction is taken into account the loading-unloading path is reduced to a set of *loading paths* (see Fig.4). So called *loading paths* and incremental volumetric strain produce the dilatancy characteristics. Figure 5 illustrates experimentally observed dilatancy characteristics. Comparison of predicted (based on Eqs.5 & 6) and experimental data is given in Fig.6.

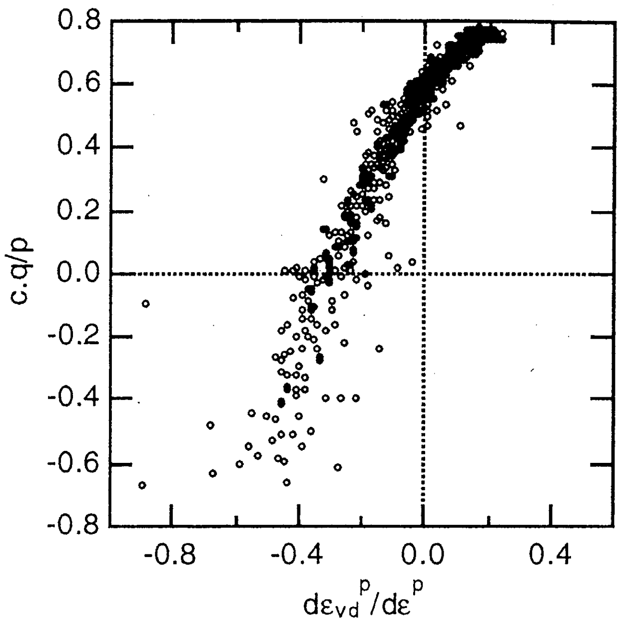


Figure 5: Stress-dilatancy relationship given by experimental data.

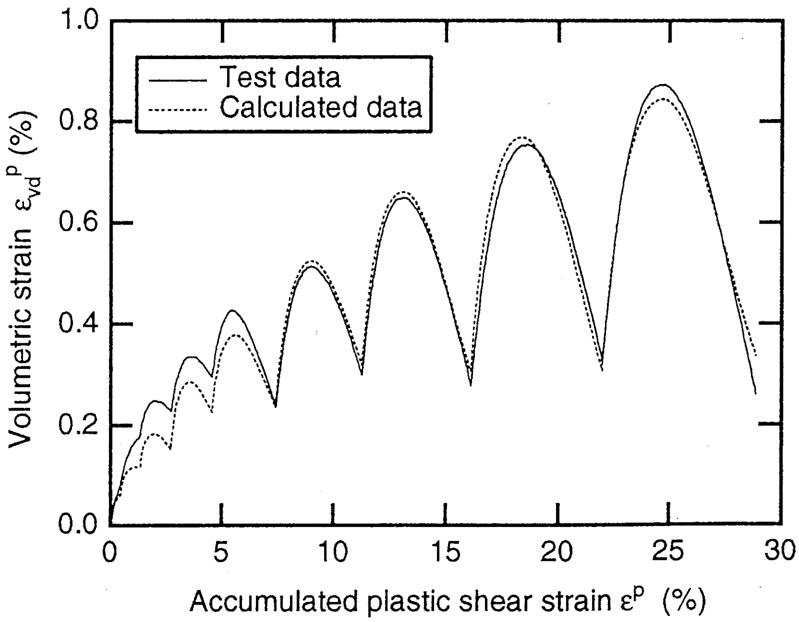


Figure 6: Estimated and observed plastic volumetric strain due to shear.

**3.4 Undrained condition.**

As far as the liquefaction is concerned the behavior under undrained condition is of prime important. Therefore, undrained cyclic behavior of Toyoura sand under torsional simple shear is examined. Test results are shown typically in Figs.7 and 8. Cyclic mobility is observed as the onset of initial liquefaction.

It is common to derive the stress-dilatancy relationship for drained test since the volume change due to shearing can be directly measured in these tests. Since it has been observed that the stress-dilatancy relation can be used in the drained test along with some modification for cyclic loading, the same relationship (given by Eq.6) is employed in undrained cyclic loading.

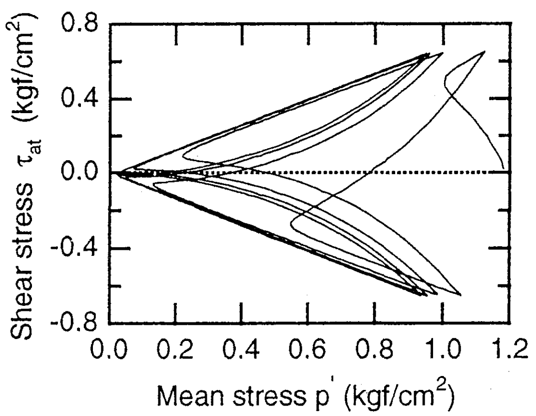


Figure 7: Stress path of undrained torsional simple shear test.

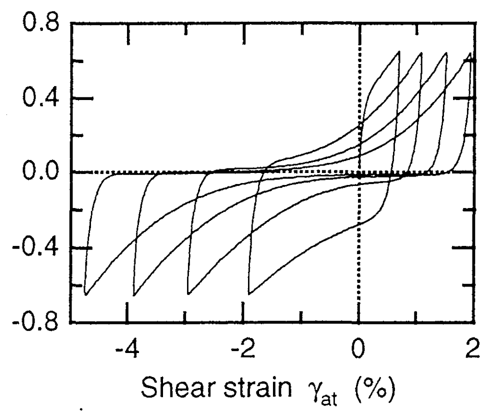


Figure 8: Stress-strain relationship for undrained torsional simple shear test.

Thus, volumetric strain due to dilatancy can be computed from Eq.6. Under the undrained condition, however this volume change does not occur, but effective mean stress changes by the amount

$$dp' = K(-d\varepsilon_{vd}^{p}) \tag{7}$$

where K is the bulk modules.

On the other hand, conceptual change of volumetric strain due to change of effective mean stress ( or volumetric strain due to consolidation) is given by

$$d\varepsilon_{vc} = -d\varepsilon_{vd.} \quad (\because d\varepsilon_{v} = d\varepsilon_{vc} + d\varepsilon_{vd} = 0\ ) \tag{8}$$

Figure 9 illustrates the variation of volumetric strain due to consolidation (given by Eq.8) with the change of mean effective stress as obtained from the torsional simple shear test data. In the same figure conventional consolidation characteristics are given by a dotted line.

It is interesting to note that variation of volumetric strain due to consolidation at low effective confining pressure is very much differ from the conventional consolidation characteristics, but very much in comply with the consolidation and shear deformation characteristics of the sand after different degree of liquefaction (FL) as reported by Yoshida et al. (1994a) and Yasuda et al. (1994). However, consolidation characteristics of sand at liquefaction is similar to conventional consolidation characteristics as implied by the gradient of solid and dotted lines at stress levels other than low effective confining stresses close to zero, in Fig. 9. Thus indicating the possibility of employing conventional liquefaction analysis at stress levels other than low effective confining stresses close to zero.

## 4 Concluding remarks

Investigation on behavior after liquefaction indicates that the use of new expression for bulk modules instead of conventional expression of bulk modules is necessary at low confining effective stress levels close to zero. This is well observed in Fig. 1 where consolidation characteristics of sand after liquefaction is illustrated.

Undrained test data produce the conceptual volumetric strain due to change of effective confining stress which is very much differ from conventional consolidation characteristics. On the other hand, the conceptual volumetric strain calculated as above show somewhat similar relation to the consolidation characteristics of sand after liquefaction. Further, the undrained test includes the cyclic mobility which can be considered as the onset of initial liquefaction or the

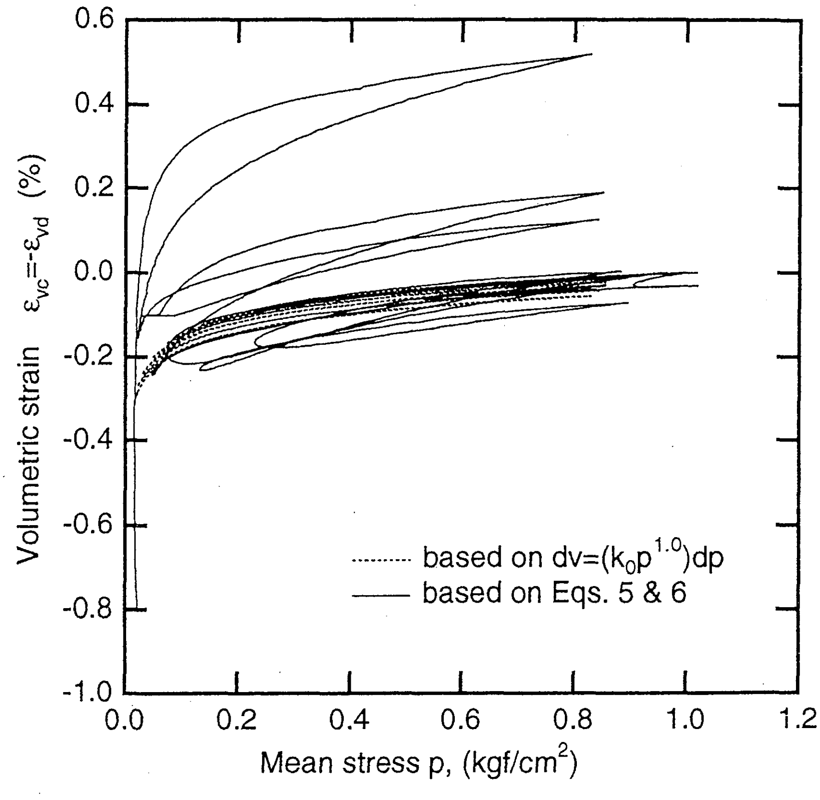


Figure 9: Volumetric strain characteristics during the on set of cyclic mobility, estimated assuming the stress-dilatancy relation given by Eq. 5 & 6.

transient to liquefaction. In deriving the conceptual volumetric strain in undrained test data, stress-dilatancy relation established and verified by drained test result is used.

In conclusion therefore, it can be deduced that behavior observed at transient to liquefaction and after liquefaction is continuos process. In other words, the behavior continuously change from cyclic loading including on set of liquefaction to behavior after liquefaction.

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