# Change of Volumetric Strain Characteristics of Sand during Cyclic Loading

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### Abstract

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Volume change characteristics of sand under cyclic loading and at large strains after liquefaction is investigated. Series of torsional shear test data and torsional simple shear test data, including both monotonic and cyclic loading, have been processed. First, stress-dilatancy relationship is developed using drained test results based on normalized shear work. Volumetric strain characteristics in wide range of confining pressure including low confining pressure levels are then investigated using undrained test data and stress-dilatancy model established using drained test data. It is shown that volumetric strain characteristics are similar to the one after liquefaction, except that the range of low stiffness region is smaller. This indicates the possible shortcomings of conventional constitutive model in simulating the behavior of sand at low confining pressure.

### **1. Introduction**

Volumetric strain characteristics play an important role in the liquefaction behavior of sand. Many researches have been conducted to develop stress-strain models (Dafalias, 1994; Towhata, 1989). Yoshida (1991) pointed out that many constitutive models based on the plasticity theory have its own difficulties in expressing the behavior of sand during and after the liquefaction. One is that, in the model, hysteretic behavior becomes stable fairly quickly after phase transformation whereas in the laboratory test, shear strain increases with loading. The other is that volume change due to dissipation of excess pore water pressure derived from models is hardly affected by loading cycles & much smaller than that of test in which volume change depends on the loading applied after liquefaction (Nagase et al., 1988).

Yasuda et al. (1994) conducted laboratory test in which sand is subjected to monotonically increasing strain after liquefaction. Yoshida et al. (1994a) processed this test data and found the existence of an unstable region near the liquefaction, the extent of this region depends on the loading cycles. Their investigation is focused on the behavior after liquefaction, and transient phenomena at liquefaction was not clear.

In this paper, the development of volumetric strain during the transient at liquefaction is investigated based on the cyclic test results. It has been found that the volume change characteristics vary continuously from transient to the behavior after liquefaction.

# 2. Behavior after liquefaction

Yoshida et al. (1994b) measured volume change due to change in effective confining pressure, in the process of excess pore water pressure dissipation after liquefaction, as shown in Fig.1. Occurrence of large volume change is observed just at the beginning of drainage, and its amount depends on the extent of loading applied after liquefaction. It has also been pointed out that in the conventional expression of bulk modules in which bulk modules is proportional to some power of effective confining pressure can express the behavior at higher effective confining pressure, but cannot express the behavior at low effective confining pressure close to zero. Yoshida and Finn (1994) proposed an equation expressing this behavior in the form,

$$\frac{p'}{p'_0} = \frac{e^{\frac{\varepsilon_v}{c}} - 1}{e^{\frac{\varepsilon_{v_0}}{c}} - 1}$$

(1)

where c is a constant and  $\varepsilon_{v0}$  is volumetric strain at the reference mean stress. As shown in Fig. 1, agreement between test result and Eq. 1 is good.

Yasuda et al. (1994) conducted laboratory test of sand in which cyclic shear load is applied first even after the onset of liquefaction and then shear strain is increased monotonically. Figure 2 shows the stress-strain relationship for the monotonic pat of the test. It has been noted that there appears a low stiffness region at the beginning of monotonic loading.  $F_L$  (Safety factor against liquefaction) value is used as an index of loading after liquefaction;  $F_L$  value decreases as the amount of loading after liquefaction increases. The range of low stiffness region depends on  $F_L$  value as seen in Fig.2. Yoshida et al. (1994a) analyzed this test and showed that the behavior can be simulated if shear modules at small strain is deduced by an equation similar to Eq. 1. The analysis is shown in Fig. 2 as well. A good agreement between the test and the analysis is observed. In addition, Yoshida et al. (1994a) pointed out that various material properties such as internal friction angle change with the amount of load applied after liquefaction.





Figure 1: Volumetric strain characteristics during the excess pore water pressure dissipation after liquefaction.

Figure 2: Stress-strain relationship of sand subjected to monotonic loading after liquefaction.

(5)

In summary, it can be noted that the material properties of liquefied soil changes due to the disturbance occurred in structure of soil skeleton at very small inter granular forces resulted by liquefaction. At low confining pressure, a new structure is yet to be developed, therefore stiffness is very small. When confining pressure increases, new structure is formed. Conventional theory seemed to be based on latter stage where material behavior is somewhat stable.

### **3. Behavior during cyclic loading Stress & Strain parameters**

$$p = (\sigma'_{a} + \sigma'_{i})/2 \qquad ; \qquad d\varepsilon_{v} = d\varepsilon_{a} + d\varepsilon_{t}$$

$$q = \sqrt{\left[(\sigma'_{a} - \sigma'_{i})/2\right]^{2} + {\sigma'_{at}}^{2}} \qquad ; \qquad d\overline{\varepsilon} = \sqrt{(d\varepsilon_{a} - d\varepsilon_{t})^{2} + (2d\varepsilon_{at})^{2}}$$

$$\tan 2\beta_{\sigma} = 2\sigma'_{at}/(\sigma'_{a} - \sigma'_{t}) \qquad ; \qquad \tan 2\beta_{d\varepsilon} = 2d\varepsilon_{at}^{p}/(d\varepsilon_{a}^{p} - d\varepsilon_{t}^{p}) \qquad (2)$$

where  $\beta_{\sigma}$  and  $\beta_{d\varepsilon}$  are the directions made by principal stress and principal strain increment direction with vertical respectively.

#### **Stress-Dilatancy relationship**

Consider the plastic shear work which is used as the primary form of stress dilatancy relation;

$$dW^p = \sigma_{ii} d\varepsilon_{ii}^p$$

In two dimensional form;

$$dW^{p} = \sigma_{ij} d\varepsilon_{ij}^{p} = \sigma_{a} d\varepsilon_{a}^{p} + \sigma_{t} d\varepsilon_{t}^{p} + 2\sigma_{at} d\varepsilon_{at}^{p}$$
(3)  
terms of stress and strain invariant as given in Eq. 2

In terms of stress and strain invariant as given in Eq. 2

$$dW^{p} = pd\varepsilon_{vd}^{p} + qd\overline{\varepsilon}^{p}\cos(2\beta_{\sigma} - 2\beta_{d\varepsilon}) = pd\varepsilon_{vd}^{p} + qd\overline{\varepsilon}^{p}\cos2\psi$$
(4)

where  $\psi = \beta_{\sigma} - \beta_{d\varepsilon}$  is the angle of non-coaxiality; the angle by which the principal stress direction and principal strain direction vary (Gutierrez, 1989).  $\varepsilon_{vd}$  is the volumetric strain due to shearing. Superscript 'p' stand for plastic component of the corresponding strain.

The stress dilatancy relationship is derived assuming that the normalized shear work as a function of accumulated plastic shear strain,

$$d\Omega^{p} = dW^{p} / p = d\varepsilon_{vd}^{p} + (q / p)d\overline{\varepsilon}^{p} \cos 2\psi = \mu d\overline{\varepsilon}^{p}$$
  
Yielding,  
$$d\varepsilon_{vd}^{p} / d\overline{\varepsilon}^{p} = \mu - c.q / p$$

where  $c = \cos 2\psi$ ,  $\mu = \mu$ ; sine of phase transformation angle) is assumed to be a constant

#### Drained condition.

Peiris et al. (1994) has noted the necessity in modifying the dilatancy relation given by Eq.5, in estimating the volumetric strain accurately. Modification for the stress-dilatancy relationship is mainly by assuming a varying  $\mu$ , given as follows.

Where  $(c \cdot q / p)_i$  is the value of  $(c \cdot q / p)$  at the beginning of each loading step. Constant  $\mu(=\mu_c)$  is assumed during virgin and/or monotonic loading.

#### Undrained condition.

As far as the liquefaction is concerned the behavior under undrained condition is of prime important. Therefore, undrained cyclic behavior of Toyoura sand under torsional simple shear is examined. Test results by Prdhan et al. (1989), are shown typically in Figs.3 and 4. Cyclic mobility is observed as the onset of initial liquefaction.

It is common to derive the stress-dilatancy relationship for drained test since the volume change due to shearing can be directly measured in these tests. Since it has been observed that the stress-dilatancy relation can be used in the drained test along with some modification for cyclic loading, the same relationship (given by Eq.6) is employed in undrained cyclic loading.



torsional simple shear test.

Figure 4: Stress-strain relationship for undrained torsional simple shear test.

Thus, volumetric strain due to dilatancy can be computed from Eq.6. Under the undrained condition, however this volume change does not occur, but effective mean stress changes by the amount

$$dp' = K(-d\varepsilon_{vd}^p) \tag{7}$$

where K is the tangent bulk modules.

On the other hand, conceptual change of volumetric strain due to change of effective mean stress ( or volumetric strain due to consolidation) is given by

$$d\varepsilon_{vc} = -d\varepsilon_{vd} \quad (\because d\varepsilon_v = d\varepsilon_{vc} + d\varepsilon_{vd} = 0)$$
(8)

Figure 5 illustrates the variation of volumetric strain due to consolidation (given by Eq.8) with the change of mean effective stress as obtained from the torsional simple shear test data. In the same figure volumetric strain derived from conventional consolidation characteristics are given by a dotted line.

It is interesting to note that variation of volumetric strain due to consolidation at low effective confining pressure is very much differ from the conventional consolidation characteristics, but very much in comply with the consolidation and shear deformation characteristics of the sand after different degree of liquefaction (FL) as reported by Yoshida et al. (1994a) and Yasuda et al. (1994). However, consolidation characteristics of sand at liquefaction is similar to conventional consolidation characteristics as implied by the gradient of solid and dotted lines at stress levels other than low effective confining stresses close to zero, (see Fig. 5). Thus indicating the limitations of conventional liquefaction analysis.

### 4. Concluding remarks

Investigation on the behavior after liquefaction indicates that the use of a new expression for bulk modules instead of conventional expression of bulk modules is necessary at low confining effective stress levels close to zero. This is well observed in Fig. 1 where consolidation characteristics of sand after liquefaction is illustrated.

Undrained test data produce the conceptual volumetric strain due to change of effective confining stress which is very much differ from conventional consolidation characteristics. On the other hand, the conceptual volumetric strain calculated as above show somewhat similar relation to the consolidation characteristics of sand after liquefaction. Further, the undrained test includes the cyclic mobility which can be considered as the onset of initial liquefaction or the transient to liquefaction. In deriving the conceptual volumetric strain in undrained test data, stress-dilatancy relation established and verified by drained test result is used.



Figure 9: Volumetric strain characteristics during the on set of cyclic mobility estimated assuming the stress-dilatancy relation given by Eq. 5 & 6.

In conclusion therefore, it can be deduced the behavior of soil changes continuously from cyclic loading including onset of liquefaction to behavior after liquefaction.

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