

## Processing of strain dependent characteristics of soil for nonlinear analysis

Nozomu Yoshida

Engineering Research Institute, Sato Kogyo Co., Ltd, Tokyo, Japan

**ABSTRACT:** Discussed are effectiveness of  $G$ - $\gamma$  and  $h$ - $\gamma$  relationships as expression of nonlinear behavior of soil for the nonlinear dynamic response analysis. Various dynamic deformation test result and numerical analysis are used for discussion. Since  $G$ - $\gamma$  and  $h$ - $\gamma$  relationships are computed based on the stable hysteresis loop, it does not include information at transient state. Loading under undrained condition allows excess pore water generation in the case of sand. This indicates that  $G$ - $\gamma$  and  $h$ - $\gamma$  relationships depend on the applied load, therefore cannot be considered as intrinsic parameter of soil. Hysteresis loop cannot be explained by a single parameter, damping ratio. Considering these, it can be considered that  $G$ - $\gamma$  and  $h$ - $\gamma$  relationships may be a good index to express nonlinear characteristics of soil, but does not include sufficient information for nonlinear dynamic response analysis, a theoretically accurate method.

### 1 INTRODUCTION

Since soil behaves in a nonlinear manner even at small strains, it is necessary to grasp nonlinear characteristics of soil and to model it relevantly so as to predict accurately the behavior of ground due to strong earthquake. A commonly used expression of the nonlinear deformation characteristics is so called  $G$ - $\gamma$  and  $h$ - $\gamma$  relationships, in which strain dependent characteristics of soil is defined by shear modulus and damping ratio.

The method to process dynamic deformation test result into  $G$ - $\gamma$  and  $h$ - $\gamma$  relationships was first conducted by Hardin and Drnevich (1972a,b). After that, these relationships have been used to express strain dependent characteristics of soil.

Nonlinear analysis of a level ground was to be used in practical by an equivalent linear method (Schnabel et al, 1972, for example) at that time, and a nonlinear method, which solves equation of motion through step-by-step time integration following the nonlinear stress-strain relationships, and therefore a theoretically precise method was not in practical use.

Although equivalent linear method takes into account the effect of strain dependent shear modulus and damping characteristic of sand, change of material property during an earthquake cannot be

considered. Namely, material property is kept constant during the earthquake. In addition, in the frequency domain analysis employed in SHAKE, energy absorbed by the nonlinear behavior of soil is considered by making a phase difference between the velocity and displacement. Therefore, hysteresis loop is an elliptic shape, whose behavior is completely described by a single parameter such as damping ratio.

The expression of nonlinear characteristics of soil by  $G$ - $\gamma$  and  $h$ - $\gamma$  relationships along with equivalent linear method is a good technique, because material property is expressed as a function with respect to single variable, shear strain. Therefore, as equivalent linear method has been becoming a mainstream of the earthquake response analysis of ground, use of  $G$ - $\gamma$  and  $h$ - $\gamma$  relationships has also become mainstream as strain dependent characteristics of soil.

Hardin and Drnevich (1972a,b) considered shear modulus and damping ratio as critical soil parameters at the time, but they did not think that this expression is complete or general to express soil behavior in dynamic problem. It is obvious that  $G$ - $\gamma$  and  $h$ - $\gamma$  relationships cannot explain the whole behavior of soil during the earthquake because they express a kind of stable state under constant amplitude loading; transient behavior cannot be

considered in this expression. This indicates that they may not give sufficient information required in the step-by-step nonlinear analysis. However,  $G$ - $\gamma$  and  $h$ - $\gamma$  relationships is still used in nonlinear characteristics of soils as target characteristics for determining the value of parameters of constitutive model, even when a precise method, nonlinear analysis, is going to be in practice, and are now going to be a standard (JSSMFE, 1993).

In this paper, we show various problems by processing the nonlinear characteristics of soils into  $G$ - $\gamma$  and  $h$ - $\gamma$  relationships, and discuss an expression of nonlinear characteristics for the step-by-step nonlinear earthquake response analysis.

## 2 DYNAMIC DEFORMATION TEST

Test for obtaining the  $G$ - $\gamma$  and  $h$ - $\gamma$  relationships is conducted by following the procedure schematically shown in Fig. 1. Either triaxial test or torsional shear test apparatus is used. Test specimen is first consolidated isotropically. Then it is subjected to a constant amplitude cyclic load under undrained condition. After certain cycles (e.g., 11 cycles) of loading, shear modulus  $G$  and damping ratio  $h$  are computed from the hysteresis loop near the end of the loading (e.g., 10th cycle). If pore water pressure generates during the loading in each loading stage, it is dissipated at the end of the stage. Then, in the next stage, the same procedure is repeated by increasing the stress amplitude.

In the followings, we call this test procedure as conventional test.

## 3 CONSTANT AMPLITUDE LOADING

In the conventional test,  $G$ - $\gamma$  and  $h$ - $\gamma$  relationships are computed from the hysteresis curve where hysteresis curve becomes to nearly stable. Conversely, stress-strain curve is not stable at the early stage of loading. However, transient state is not considered in the expression by  $G$ - $\gamma$  and  $h$ - $\gamma$  relationships. Figure 2 shows an example of dynamic deformation test by means of torsional shear test apparatus (Yamashita, 1994). Here, plotted are not only  $G$  and  $h$  computed from the 10th cycle of loading, but also those at the first and second cycles. It is clear that  $G$ - $\gamma$  and  $h$ - $\gamma$  relationships changes much at the beginning of loading in each stage, and gradually becomes stable. The difference of the relationships at the first and 10th cycles is small at small strains, but becomes very large at large strains. Especially, difference of

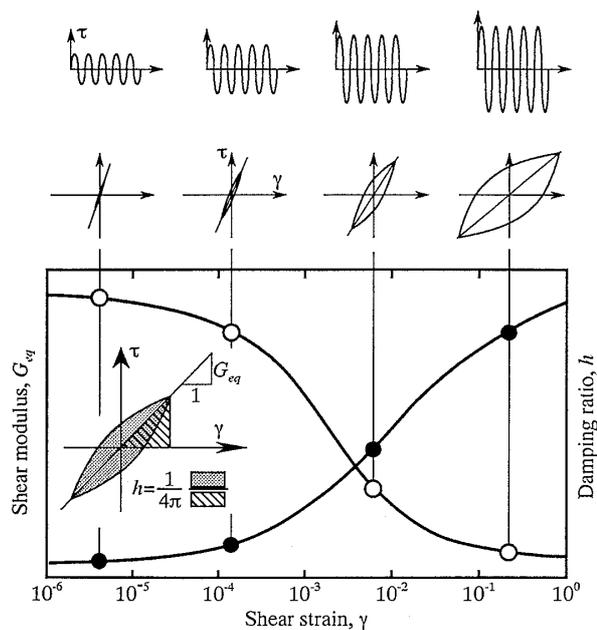


Fig. 1 Schematic figure showing the procedure to obtain  $G$ - $\gamma$  and  $h$ - $\gamma$  relationships

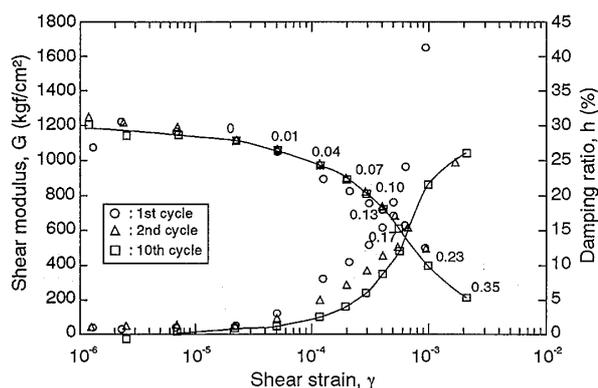


Fig. 2  $G$ - $\gamma$  and  $h$ - $\gamma$  relationships

damping ratio is large.

Generally speaking, peak response such as peak acceleration occurs when strain is the largest. In other words, important part in the stress-strain curve may not be the behavior at a stable state, but the one at virgin loading.

After peak response, strain decreases from maximum value. However, since test is conducted by increasing stress amplitude, behavior with decreasing strain amplitude is not known.

## 4 DRAINAGE CONDITION

At the beginning of the research in this field, test was conducted under drained conditions in the case

of sand (Iwasaki, 1972, for example). Although behavior of soil skeleton or dry sand is obtained by test under undrained condition, behavior of sand under the water table, which is a more general situation in the ordinary ground, is not obtained, because pore water cannot move freely in the pore. Therefore, test data may be applicable for modeling the behavior of soil for effective stress analysis, but not for total stress analysis. Note that since volume change or excess pore water generation is not measured,  $G-\gamma$  and  $h-\gamma$  relationships are not sufficient for effective stress analysis.

So as to overcome this shortage, test condition was changed from drained condition to undrained condition (Kokusho, 1980, for example), and it now is going to be a standard method. On the background of this change, undrained condition is supposed to hold during the earthquake, because duration of earthquake is short for pore water to dissipate outside the soil element. Test under undrained condition has another good point; test can be conducted under the same procedure from clay to sand and even gravel. There is no point to doubt about the effectiveness of undrained condition for clay. For sand, however, it is sure that undrained condition holds as an approximation, but it cannot hold exactly during the earthquake even if duration of the earthquake is short.

For example, Fig. 3 shows pore water pressure distribution of a level ground analyzed under drained and undrained condition. Result of analyses under drained condition shows very smooth distribution because of the interaction with permeable layer. The difference of excess pore water pressure between drained and undrained conditions are not small, which indicates that material property evaluated under undrained condition has certain amount of error.

Under the undrained condition, excess pore water pressure may generate in silt to gravel. For example, numerals in Fig. 2 are excess pore water pressure ratio generated in each stage. Note that excess pore water pressure is dissipated at the end of each stage. Therefore, fairly good amount of excess pore water pressure has generated until the end of loading.

Amount of excess pore water pressure generation will change the  $G-\gamma$  and  $h-\gamma$  relationships. It depends on how much excess pore water pressure is generated. This indicates that the same  $G-\gamma$  and  $h-\gamma$  relationships cannot be used for the analyses with different earthquakes. It is well known that duration of ground shaking is longer at big earthquake than at

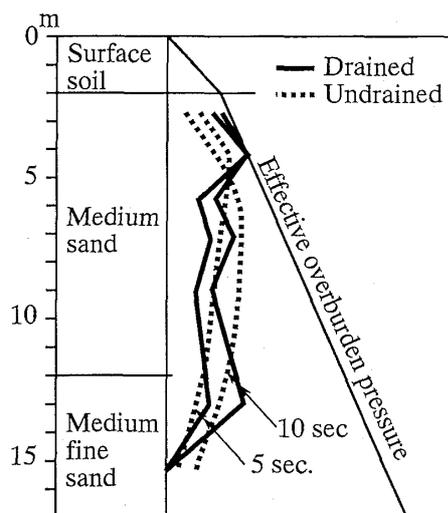


Fig. 3 Pore water pressure distribution computed under undrained and drained conditions (Kawagishicho site, Niigata earthquake).

small earthquake. Therefore, if we compare the response at a site with longer epicentral distance at the time of big earthquake with the one at a site with shorter distance at the time of medium earthquake, former site will be subjected to larger number of cycles than the latter even if peak response value may be the same.

## 5 EFFECTIVE DAMPING RATIO

Hysteretic behavior is expressed only by single parameter, damping ratio. Definition of damping ratio is shown in Fig. 1. It is a ratio of absorbed energy to strain energy. Therefore, actual energy absorption is accurate only when area of hysteresis loop and strain energy are evaluated correctly. In other words, if evaluation of strain energy or  $G-\gamma$  curve have an error in the constitutive model, which frequently occurs, use of damping ratio expressed by  $h$  does not offer actual energy absorption due to hysteretic damping.

Another problem expressing the hysteresis loop by  $h$  is that it is impossible to distinguish the shape of hysteresis loop. In the equivalent linear analysis, shape of hysteresis loop is elliptic, therefore can be expressed by single parameter. However, in the nonlinear analysis, we should trace actual stress path; the information of the hysteresis loop as damping ratio is not sufficient. Fig. 4(a) shows  $G-\gamma$  and  $h-\gamma$  relationships of silty sand, for example, and Fig. 4(b) shows shape of hysteresis loop from which shear modulus and damping ratio are computed at particular stage A to D. It is noted that, from C to D,

damping ratio decreases as shear stress amplitude increases. The reason is clear when looking at Fig. 4(b)D; the shape of hysteresis loop is inverse S-shape. This is obviously caused by the excess pore water pressure generation under undrained condition. Elliptic shape expression of hysteresis loop may be justified in stages A and B. The shape gradually changes as excess pore water pressure generates. The elliptic expression may not be good in stage C, and probably, it is not good at stage D.

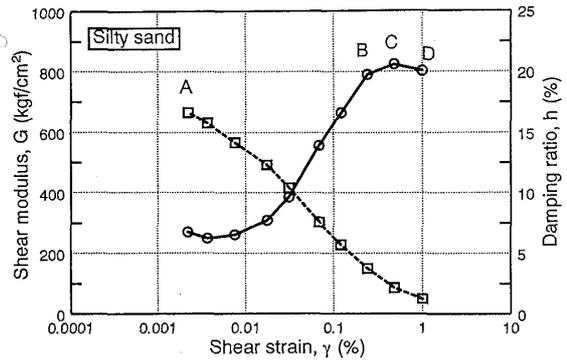
In summary, expression of hysteresis loop by damping ratio has difficulties in expressing the nonlinear behaviors of sand at least on two points. One is the shape of hysteresis loops. The other is that  $G$ - $\gamma$  relationships must be accurate to obtain accurate energy absorption due to hysteretic behavior from damping ratio.

### 6 EFFECT OF INITIAL STRESS

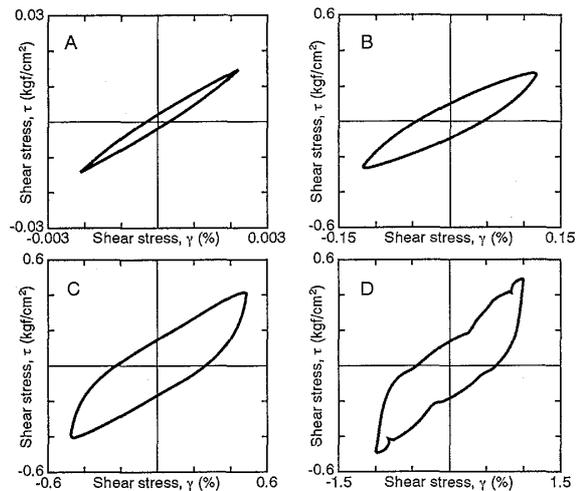
Soil is usually under an anisotropically consolidated state even in a level ground. This indicates that soil is already subjected to the shear stress before the earthquake. On the other hand, test specimen used in the conventional test is consolidated isotropically, therefore, not subjected to initial shear stress.

Figure 5 shows Mohr's circle and stress-strain curve of soils under isotropically and anisotropically consolidated states subjected to cyclic load schematically. Initial stress state is expressed as a point in the isotropically consolidated soil. Under the constant stress amplitude loading, Mohr's circle changes from a point corresponding to the initial state to a circle whose radius equals to the shear stress amplitude, which region is shown by shaded. As seen in the figure, stress-strain curve is symmetric with respect to the origin.

On the other hand, in the case of soils loaded from an anisotropically consolidated states, Mohr's circle at the beginning of loading have finite radius depending on the coefficient of earth pressure at rest  $K_0$ . Under the repeated horizontal shear, therefore, it moves only outside the initial circle, which is also shown as shaded region in the figure. Because of the existence of initial shear, stiffness under virgin loading is smaller than the one of isotropically consolidated soil. As a result, strain under virgin loading is larger compared with the strain at isotropically consolidated soil. When unloading takes place, however, soils recovers its initial stiffness. As a result, strain after unloading is smaller than the strain at virgin loading. Therefore, stress-strain curve is not symmetric with respect to



(a)  $G$ - $\gamma$  and  $h$ - $\gamma$  relationships



(b) hysteresis loops; Note that origin of the figure is taken so that it comes to the center of hysteresis curve

Fig. 4 Hysteretic behavior of silty sand

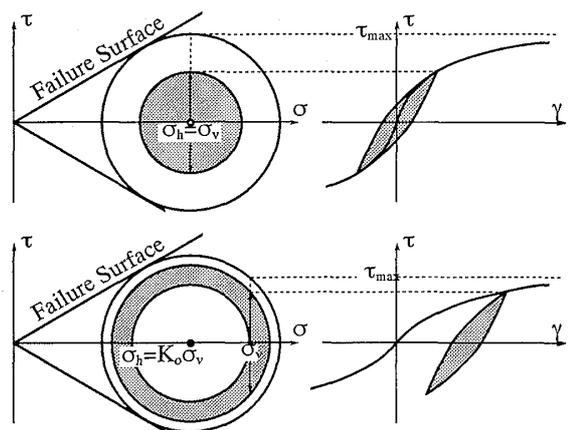


Fig. 5 Schematic figure showing the differences of the behavior of level ground with and without initial shear

the origin; drift of shear strain is strongly affected by the direction of initial loading very much.

Two stress-strain curves in Fig. 5 are quite different to each other. If the values of  $G$  and  $h$  are computed from the hysteresis loop shown in the shaded in Fig. 5, however, resultant shear modulus and damping ratio are the same, because they are computed from the shape of shaded region therefore drift of shear strain occurred in the anisotropically consolidated case is not considered.

This can be confirmed by the test, too. Figure 6 shows result of dynamic deformation test by torsional shear test, in which initial stress is set so that coefficient of earth pressure at rest,  $K_0$ , is 0.5. Here, hysteresis loop at the 10th cycle in every stage is used to produce  $G-\gamma$  and  $h-\gamma$  relationships. It is clear that hysteresis loop drifts toward the direction of virgin loading in each loading stage. However, this drift is not considered in the  $G-\gamma$  and  $h-\gamma$  relationships.

Dynamic response analysis is conducted to investigate the effect of initial stress consideration. Fill land at Tokyo Bay area, where detailed soil data is obtained (Masuda et al., 1994), is analyzed by nonlinear step by step analysis code STADAS (Yoshida, 1994). Stress-strain relationships proposed by Yoshida and Tsujino (1989) was employed in the analysis, in which stress-strain curve for virgin loading is expressed as piecewise linear function whose secant shear modulus coincide with specified value, and hyperbolic model is used for hysteresis whose backbone curve is chosen so that hysteretic damping is equals to specified damping characteristics. Therefore, this model completely agrees with given  $G-\gamma$  and  $h-\gamma$  relationships. Two dimensional analysis is conducted so that anisotropic stress state can be considered. The analysis starts with isotropic or anisotropic ( $K_0=0.7$ ) stress state. Figure 7 shows soil profiles and peak response values, and Fig. 8 shows stress-strain relationships at 9th layer where

nonlinear behavior is predominant.

Analysis starting from isotropic consolidation gives larger acceleration and shear stress, but smaller displacement and shear strain compared with the analysis under an isotropic consolidation. The reason of this difference is clear by looking at Fig. 8. Shear stiffness and strains is much smaller in the isotropic consolidation case than those in the anisotropic consolidation case. Difference between

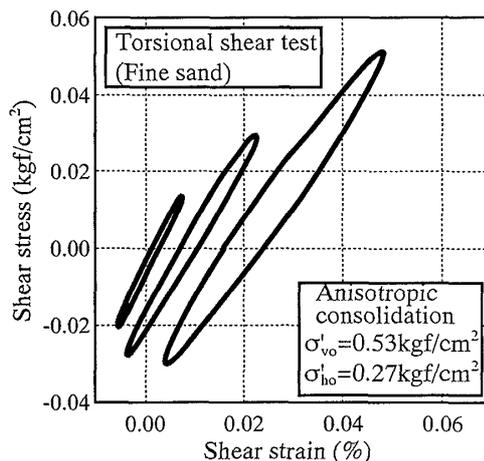


Fig. 6 Stress-strain relationships of Toyoura sand subjected from anisotropically consolidated state; hysteresis loop drift the direction of virgin loading, which is not considered in  $G-\gamma$  and  $h-\gamma$  relationships

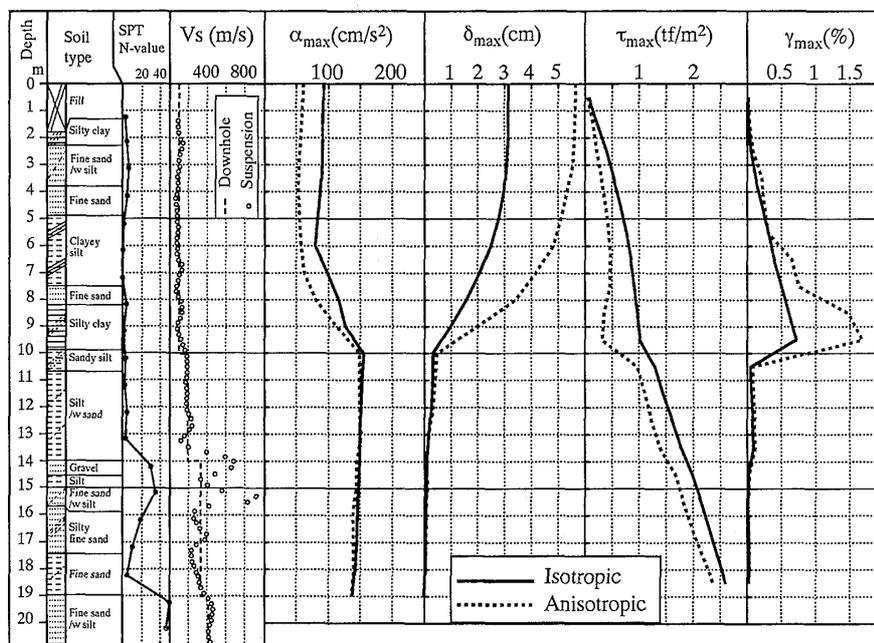


Fig. 7 Soils profiles and peak response values to investigate the effect of anisotropic consolidation

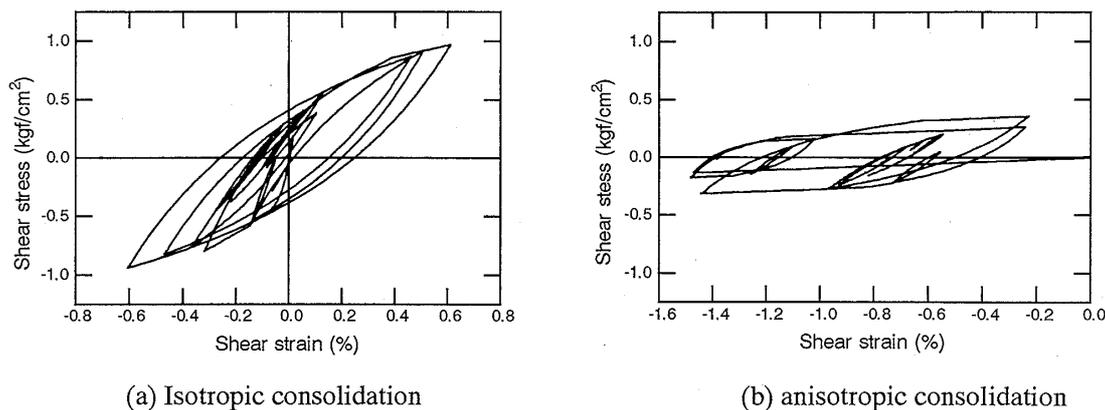


Fig. 8 Stress-strain relationships at 9th layer

these two analyses is not small, therefore initial stress condition cannot be neglected.

## 7 CONCLUDING REMARKS

In this paper, we discussed applicability of  $G-\gamma$  and  $h-\gamma$  relationships as nonlinear characteristics of sand to be used in the nonlinear dynamic response analysis. At present, equivalent linear analysis is the mainstream in the field of dynamic response analysis. Equivalent linear method is an approximate method to consider nonlinear behavior of soil. It has various limitation in considering nonlinear property. One of the key limitation is that it cannot consider transient behavior; material property is kept constant during earthquake. This limitation fit  $G-\gamma$  and  $h-\gamma$  expression. This seems to be the reason why  $G-\gamma$  and  $h-\gamma$  relationships has become mainstream or even a standard to express nonlinear behavior of soil.

Since equivalent linear method is an approximation, the analysis should move towards nonlinear analysis in which change of material property is traced in the time marching analysis. However, as discussed in this paper, the expression of nonlinear behavior by  $G-\gamma$  and  $h-\gamma$  relationships has various problems. If employed stress-strain model is not suitable, we cannot expect accurate prediction even if nonlinear method is employed. At present, we can say that  $G-\gamma$  and  $h-\gamma$  relationships obtained by the dynamic deformation test is an index or characteristic value, but do not have sufficient information for nonlinear analysis. A new expression is required instead of  $G-\gamma$  and  $h-\gamma$  relationships for better prediction of the behavior of ground during earthquake.

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